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AN INTRODUCTION TO LOGIC

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AN INTRODUCTION TO LOGIC

FROM THE STANDPOINT OF EDUCATION

BY

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PREFACE

This book is designed for those whose concern is with Education. It describes the main aspects of the thinking process with the view of giving the teacher a clear idea of the fundamental things to aim at in getting the child to think.

Certain features perhaps call for a word of explanation. I have endeavoured to begin at the standpoint of the person who is ignorant of Logic, and to familiarise him with the thinking process before proceeding to analyse it. In so doing, I have had in view the method of teaching by which the student is led to construct his own science by his own efforts, in discussion with the teacher.

It follows that the exercises form an integral part of the text. They deal as far as possible with problems which arise in school work. They are intended, of course, to test the student's knowledge; but their main purpose is to suggest topics for discussion between the teacher and his pupils. Hence they contain matter which would usually have been treated in the text. But in my opinion the text itself represents results to be gained by the student in discussion, rather than points to be put to him in monologic exposition.

The fundamental principle of all formal sciences is that of reduction to elementary type forms ; and this was the basis of Aristotle's treatment of the syllogism. It is the principle which must be used in any serious treatment, and is adopted here. But the four forms chosen as elementary by Aristotle made reduction a difficult process, and caused the introduction of various devices which tended to obscure that principle. The type syllogisms used in this book are a combination of Aristotle's first and second figures, and make reduction a very simple matter. They follow naturally from the meanings of the universal affirmative and the universal negative propositions. They can be still further reduced to two, one first figure and one second figure syllogism. But it seems preferable, for many reasons, to keep the four forms.

As a result of adopting this method, the cumbrous rules of the syllogism, and the whole account of distribution of terms, can be omitted without any loss of accuracy or precision. Conversion and obversion can be introduced at a point at which they are seen to be necessary if progress is to be made. Apart from this, there is no discussion of immediate inference. However interesting such inferences may be to the professed logician, it seems unnecessary to discuss them exhaustively in such a book as this. For similar reasons I have not entered upon a technical discussion of fallacies, nor upon the history of Logic. Indeed, the object has been throughout to give an effective minimum.

I do not suggest that no historical facts be introduced. The teacher himself will know how to

stimulate interest by a judicious introduction of such historical matter as his students are ready to appreciate. He can gauge interests and answer questions: the printed page, when you question it, "always returns the same answer."

It is a pleasure to acknowledge the very great debt which the book owes to the willing suggestions and criticisms of many colleagues and friends.

LEONARD J. RUSSELL.

THE UNIVERSITY,
GLASGOW, *December*, 1913.

CONTENTS

CHAPTER I.

THE PROCESSES INVOLVED IN ACTION.

As I am sitting before the fire reading my paper, I hear the clock strike. It is half-past eight and I must start for my train, for I must be at my place of business by nine. I look at the glass and at the clouds. I put on my hat and coat, take my umbrella, and start for the station. When near the station, I see that the train is signalled, and a moment later see a white line of smoke and then the engine. I hurry, for I know that the train does not stay long in the station.

When I am comfortably seated in the carriage, I reflect on what has just happened. What were the processes my mind went through when I acted as I did ? There were so many processes that I can only think of a few of them. In the first place, it is not quite true to say that I *heard* the clock strike ; I heard a particular sound, and *concluded* or *inferred* that it came from the clock, since no other thing makes just the noise I heard. There was no need for me to look at the clock, for past experience has told me that it strikes one at the half-hour, and memory told me that it was after eight. Past

experience told me also that a train runs every morning at a certain time from the station near at hand to the town in which I have my place of business, and that it arrives at a certain time. I remember also the way to the station, and that when I go out I must take my hat and coat. From the condition of the clouds and the state of the barometer, I infer that it is likely to rain ; and experience in the past tells me that it is safer to take an umbrella. I do not see the *train* signalled : I see the signal down, and past experience has shown me that when the signal is down it is a sign that the train will shortly come : thus I *infer* from the signal the approach of the train. From the white line of smoke I infer the presence of the train very near the station.

1. Let us analyse these processes a little further. In the first place, I should not have got up from the fire unless I had some definite purpose in doing so. I have, in fact, the object in view of getting to my business at a certain time. That is connected with very many other purposes, wherein are involved many other people, all over the world. I have to earn my living, and my business enables me to do this because of the way in which commerce is organised. Thus, *e.g.* I may go into an office and do nothing but write : and this enables me to obtain bread, clothes, a house, etc. Not that it would have done so in other days. *E.g.* if I had lived a thousand years ago, the connection of my work with my bread would have been much closer. I might have been tending the herds or ploughing the fields.

2. This end involves means : and my ability to

provide means to my end depends on the extent to which I can anticipate what will happen if I take certain steps. I have gradually learned by experience many things about the life round me, and memory brings them to my aid now. Take the striking of the clock. When I was a very young child, the striking of the clock came as a surprise to me. I had seen clocks, and knew how to tell the time, but had not heard a clock strike. But one day a new clock was brought into the house, and I heard it strike. I wondered where the noise came from, and what it meant. After careful and anxious watching, I discovered that the clock struck one when the hands were at the half-hour, and in a very complicated way when they were at the hour. I thus formed the generalisation, that when the clock strikes one, the hands are at the half-hour. I was now able to use this knowledge when I heard the clock strike one: applying the rule which I had formed, I concluded that the hands stood at the half-hour. But on testing this, I discovered an exception. The clock struck one also if the hands showed one o'clock. I was thus led to alter my generalisation, so as to include this exception. So with all my other generalisations: that when the clouds are heavy it will probably rain; that when the signal is down a train is coming, etc.; these were all learned by experiencing the one thing in conjunction with the other many times, and thence forming a tentative general rule; by applying this tentative general rule (or hypothesis as it is called) to particular cases, and thus verifying or modifying it.

In this process of learning by experience you have all the stages which are gone through by the most advanced scientist in reaching his generalisations. Wherever you have it, the process is always the same, and has the same steps, of which the chief are: experience of particular facts (observation and experiment), grouping these facts together into general propositions (induction: comparison and classification, the formation of hypotheses), applying these general propositions to new particular cases (deduction), and thus, finally, verifying our general propositions.

3. This process has another aspect. My experience proceeds by means of, and results in, *concepts*. I understand much more of the significance of a newspaper, of my office, than a child does. The newspaper gives both of us, perhaps, an image of a white sheet with black marks. But it may to the young child mean nothing more than a thing which crackles wonderfully when crushed, and can be torn up into innumerable pieces. His *conception* of it (as we say in ordinary life), is not so wide as mine, which includes an idea of its significance as a bringer of news, as the product of a vast organisation reaching out to the ends of the earth. Now a concept is, as it were, a cluster of general rules round an image. We know what experiences will follow if we react in certain ways with the thing which corresponds to our image. The young child, on seeing a small white cube, knows that if he sucks it it will taste sweet. Though he could not express all it means, he has formed his concept of sugar in precisely the same way as the scientist forms his generalisations.

Thus my simple act of going to the office involves the formation (*a*) of concepts of objects, and (*b*) of general rules regarding these objects, both by means of a fourfold process ; and it involves also the use of this experience in providing means to the effecting of my end. But by this time the train has reached my destination, and my first investigation into the subject-matter of Logic is at an end.

EXERCISES.

1. Show the part played by knowledge (or ignorance) in your feelings on entering College (*a*) for the first time, (*b*) some time later.

2. Why is a town-bred person timid in dealing with cows, while a country-bred person is not ?

3. What is it that constitutes "presence of mind" ? In what situations would a good swimmer be likely to show more presence of mind than a poor one ? Why ?

4. Compare the train of ideas and actions started (*a*) in a young child, (*b*) in an adult, by hearing a clap of thunder.

5. Make a careful analysis of Robinson Crusoe's feelings and thoughts on seeing the footprint on the sand. What did he infer from the footprint ? In what way did his past experience guide him in the inferences he drew ?

6. " He saw a labourer lying on the ground asleep." How much of this is inference ?

CHAPTER II.

THE PROCESSES INVOLVED IN THINKING.

We have seen that acting in ordinary life is possible only because, at each step, we apply the generalisations which we have gradually arrived at by experience. The solution of problems which arise in ordinary life involves a similar application. Here we have thinking or reasoning made explicit. Huxley gives the following example, and his remarks will illustrate the point we desire to make: "I will suppose that one of you, on coming down in the morning to the parlour of your house, finds that a teapot and some spoons which had been left in the room on the previous evening are gone—the window is open, and you observe the mark of a dirty hand on the window-frame, and perhaps, in addition to that, you notice the impress of a hobnailed shoe on the gravel outside. All these phenomena have struck your attention instantly, and before two seconds have passed you say, 'Oh, somebody has broken open the window, entered the room, and run off with the spoons and the teapot!' That speech is out of your mouth in a moment. And you will probably add, 'I know there has; I am quite sure of it!'

You mean to say exactly what you know; but in reality you are giving expression to what is, in all essential particulars, an Hypothesis. You do not know it at all; it is nothing but an hypothesis rapidly framed in your own mind! And it is an hypothesis founded on a long train of inductions and deductions.

"What are those inductions and deductions, and how have you got at this hypothesis? You have observed, in the first place, that the window is open; but by a train of reasoning involving many inductions and deductions, you have probably arrived long before at the General Law—and a very good one it is—that windows do not open of themselves; and you therefore conclude that something has opened the window. A second general law that you have arrived at in the same way is, that teapots and spoons do not go out of the window spontaneously, and you are satisfied that, as they are not now where you left them, they have been removed. In the third place, you look at the marks on the window-sill, and the shoe marks outside, and you say that in all previous experience the former kind of mark has never been produced by anything else but the hand of a human being; and the same experience shows that no other animal but man at present wears shoes with hob-nails in them such as would produce the marks on the gravel. You have further, a general law, founded on observation and experience, and that is, I am sorry to say, a very universal and unimpeachable one, that some men are thieves; and you assume at once from all these premises—and that is what constitutes your hypothesis—that the man who

made the marks outside and on the window-sill, opened the window, got into the room, and stole your teapot and spoons. You have now arrived at a *Vera Causa*; you have assumed a Cause which it is plain is competent to produce all the phenomena you have observed; you can explain all these phenomena only by the hypothesis of a thief. But that is a hypothetical conclusion, of the justice of which you have no absolute proof at all; it is only rendered highly probable by a series of inductive and deductive reasonings.

"Now, in this supposititious case, I have taken phenomena of a very common kind, in order that you might see what are the different steps in an ordinary process of reasoning, if you will only take the trouble to analyse it carefully. All the operations I have described, you will see, are involved in the mind of any man of sense in leading him to a conclusion in this case. . . . I say you are led . . . to your conclusion by exactly the same train of reasoning as that which a man of science pursues when he is endeavouring to discover the origin and laws of the most occult phenomena. The only difference is, that the nature of the inquiry being more abstruse, every step has to be most carefully watched, so that there may not be a single crack or flaw in your hypothesis." [1]

2. If we watch the scientist at work we shall see how similar is the method he pursues. Let us take a young student in the laboratory. He has discovered by observation that iron rusts readily when moist: and the problem is, Why does it rust? Here

[1] Huxley, *Essays*, p. 193. (Everyman Ed.)

we have an investigation similar in nature to the previous one. In Huxley's example we could see at once what changes had happened in the room; in the laboratory where the changes are more subtle, the scientist has one fundamental method of investigating change, namely, weighing. Weighing tells us whether in rusting, something is added to the iron, or whether something is lost. But how shall we say what was the weight of the piece of iron, before it rusted? Only if we had taken the precaution to weigh it then. This is the essence of a scientific experiment. Let us *make* a piece of iron rust, having first weighed it. We discover that the rusty iron is heavier than the iron in its ordinary state. Thus something has been added. What? The iron was in the air; we made the iron rust by making it wet; we kept it away from dust, etc. Thus as far as we can see, water and air are the only two possible things which can have been added. Now if air was added to the iron, it must have been lost from the surrounding air; how shall we know if this is the case? We must again perform the experiment, taking another precaution. If we put moist iron filings into a definite quantity of air, so that no new air can come in and none get out, we shall know more clearly what has happened. Fortunately, we are able to do this in a very simple and beautiful way. If we hang the iron up inside an inverted jar, and place the jar with its mouth in a basin of water, the air is confined in the jar, and if any is used up, water rises to take its place. Thus we do two things at once: we confine the air in the jar, and have a very effective means of judging how much air is used up, if any is. On

making the experiment, we find that the water gradually rises in the jar; and, finally, when about one-fifth of the jar is filled with water, the rising stops. However much iron we have in the jar, only one-fifth of the air is used up. Here is a new fact: what is its significance? Suppose we take the air which is left in the jar, and put some new iron into it, will the iron rust? We find that it will not. We conclude that iron rusts because something in the air is added to it, but that not all air is active in rusting iron. Air perhaps is made up of two things, one which does, one which does not, cause iron to rust. But what of the water with which we moistened the filings? That is the subject of further experiments, in which we discover the part played by it in the rusting.[1]

3. Here we have a more complicated process, but still in essentials the same process. We have to apply the results of past generalisations in discovering the most probable cause. Although our experience does not tell us at once what has caused the rust, it does tell us to look for something which was in contact with the iron. Water and air are the two most probable agents. Here is the formation of a hypothesis. "Perhaps air did it." We must test this hypothesis. If air did it, then air is added to the iron, and hence is used up, when iron rusts. At this point the scientist interferes with nature, in order to be sure what conditions are present in his experiment. He isolates the moist iron and a definite quantity of air, and discovers that his hypothesis was correct. The repetition of the ex-

[1] Armstrong, *The Teaching of Scientific Method*, p. 371.

periment with different pieces of iron and different apparatus confirms this result, and brings to light a new fact, namely, that apparently only a part of the air is concerned in the change. But if only a part is concerned, then the remainder will not rust iron. This suggests an experiment for the sake of verifying our result—without which we should not have complete confidence in it—and on performing the experiment, our expectations are realised.

In this we have the following steps: (*a*) Past generalisations are made use of in forming probable hypotheses. (*b*) Each hypothesis in turn is tested, by deducing what would result if the hypothesis were true, and finding out whether this actually does result. This process finally leaves us with only one hypothesis. (*c*) The successful hypothesis is verified by repeated experiment and observation. Of course, in the case of an event which happened in the past, we are more restricted for the testing of our hypothesis than in the case of an event which can be made to happen at any time, such as the rusting of the iron; it is in the latter cases that experiment is possible.

4. It will already have become clear that Logic is concerned with these processes involved in thought and action. In what way does it discuss them? In all our thought we have been occupied in proving things. All our action has presupposed that certain things are admitted as proved. The child proved that when the clock struck one it was either the half-hour or one o'clock. The ordinary man accepts the fact that the signal is down as a proof that the line is clear. The scientist has proved by weighing and

by his series of experiments, that air is the chief factor in the rusting of iron. Now, as Huxley says, the only difference between the ordinary man and the scientist is that the latter examines things with greater care: the scientist demands more rigid proofs than the ordinary man. The problem is, what constitutes proof—proof such as to satisfy the most scrupulous scientist? That is the central problem of Logic.

EXERCISES.

1. Describe an instance of reasoning you have gone through in ordinary life. Bring out its various steps.

2. Describe as accurately as you can the reasoning by which you decided how to solve (*a*) a geometrical problem which was fairly familiar, (*b*) one which at first sight seemed to afford no clue.

3. We have described the experiment dealing with iron rust, in such a way as to bring out the processes involved in the search for a cause. Describe it again, from the point of view of a person *watching* the experiment. What inferences would he draw?

4. In a detective story (*e.g.* one of the Sherlock Holmes series), analyse and test the value of the evidence leading to the conclusions. Discuss whether, before the solution is given, the reader is in possession of sufficient information to enable him to find the solution for himself.

5. Discuss the meaning of the word "evidence," in relation to the drawing of conclusions (or inference). What is meant by circumstantial evidence?

6. What is inference?

SUGGESTIONS FOR READING.

Armstrong: *The Teaching of Scientific Method.* Chs. XVIII., XIX., XX.

T. H. Huxley: *Essays* (" Man's Place in Nature," etc.), Everyman Edn., Lectures IV.-X.

E. J. Russell: *Lessons on Soil.* (Cambridge Nature Study Series.)

Conan Doyle: *The Adventures of Sherlock Holmes.*

Wilkie Collins: *The Moonstone.*

G. K. Chesterton: *The Innocence of Father Brown.*

CHAPTER III.

THE GROWTH OF THE CONCEPT.

1. All thinking is based on, ~~and~~ or results in, concepts. We shall get a clearer idea of what thinking is by a short discussion of the way in which a child comes to have concepts.

A child was taken into a concert-hall where there were tip-up seats. Happening to get up, it found that the seat swung back. It pressed the seat down again, but saw that it remained down only so long as it was held, swinging back on being released. The child then noticed that all the other seats in the row were in a vertical position ; and its experience with its own seat roused its curiosity as to the others. It passed along the row, pressing down each seat in turn and letting it swing back. This having been done several times, it returned to its own seat. On inquiring what such seats were called, it was told that they were tip-up seats. The child thus formed the concept of a tip-up seat.

This is an instance of the way in which concepts are formed in general.

2. At the earliest stages, a child attends only to what arrests its attention. It moves its head to the

light, or towards a sound. As it grows older, and crawls about, its curiosity is insatiable. It handles things, experimenting with them in various ways. The brightness of the fire draws its attention: it goes nearer and (say) gets burned. After a few such experiences, the brightness becomes a warning not to go too near: thus one characteristic becomes a sign of possible experiences to be got from the object: *i.e.* of other characteristics. Similarly with articles of food: after it has handled and tasted a lump of sugar, the shape and colour of the sugar suggest or stand for its other qualities. A round shape becomes the sign of possible experiences—of rolling, throwing, etc.

As it reacts with various objects, it gradually comes to the discovery that certain of them, presenting largely the same sets of characteristics, can be depended on to act in similar ways: thus any chair can be used for climbing on. It hears things called by names, and later, calls things by names itself. Its knowledge of various objects is thus organised. Certain characteristics are central, and the rest are grouped round them. My concept of a horse is thus all that a horse means for me, organised or grouped round certain central or fundamental meanings. The more I know about horses, the more a horse means for me, the richer my concept becomes, and I become the more ready to deal successfully with the various situations in which horses play an important part. A complex concept is more useful than a vague one, because it suggests more possibilities of thought or action, in dealing with things. And it can suggest these possibilities,

just because it is organised, *i.e.* connected with the obvious and fundamental characteristics by which I recognise the thing.

3. The child's knowledge is associated with, and is fixed by means of, images ; thus its knowledge of cats is bound up with, or, we might say, grouped round, an image. The particular image it has, will depend on its knowledge, and will in turn condition its knowledge. Let us look at this. If its own cat is white, and has a long tail, then before it has seen any other cats, these characteristics will probably form part of its image. It has an image of an individual thing. If it hears of other cats, it will expect them to be white, and to have long tails. But when it comes to know other cats, the characters which at first formed part of its image of a cat, receive modification. The new image is a sort of composite photograph, emphasising those characteristics which appear the same in all the cats the child knows, and blurring those which vary in different cats. It becomes less concrete, and more general or abstract. We may have left finally only a vague representation of shape. But this vague shape still plays the part which the original image did : it is bound up with the child's whole knowledge of cats. In some cases there may be little more left than the sound of the word.

4. A town child has a fairly complex concept of a railway station, a tram-car, a large shop, a crowd of people ; and a poor concept of a farm-yard, a peewit, a tramp steamer, a gold mine.

As we have seen, these concepts have arisen gradually, through experience. The town child has

a complex concept of a railway station because he has used the station so often that he is familiar with most of the activities which go on there. His concept of a farm-yard is vague because he has not lived through many experiences there ; and thus the sight of a farm-yard, or the sounds associated with it, cannot stand for more than a very few possible experiences.

5. Now it is possible to have a very complex concept of a thing, without being able to give a detailed account of it. Because a child cannot give a clear definition of a penny, or a ship, or cruelty, it does not follow that these things have no meaning for him. But it does follow that the various elements of meaning have not yet been clearly separated out and distinguished. Thus it is quite possible that some of the meanings associated with the concept are irrelevant : as in the case of a child who thinks that a king always wears a crown, or that a poodle dog has hair only in ruffs round its neck, body, and feet. Again, so long as the meanings have not been distinctly formulated, there can be little confidence in dealing with a case which presents doubtful characteristics.

Thus for effective dealing with things, it is essential that the elements of meaning should be clearly formulated in the mind. A concept in which this is so is called a *logical concept*. In defining a word, a child will generally give a vague functional account : it will tell what the thing does, or what you do with the thing. " A nail is something to put things together." " A nut is something with a shell good to eat." " A ring is what you wear on your finger,"

"A garden is to walk in."[1] The task of leading the child from such a concept to a logical concept is one of the main tasks of the educator; logic studies the conditions which have to be fulfilled by a logical concept. That is what we are now to discuss.

EXERCISES.

1. Exhibit the various stages of the growth of your concept "college," so far as it is at present formed. State (*a*) any modification which experience has made in the tentative concept formed in expectation, (*b*) those elements which still remain mere expectations. Where did your expectations come from? What images are associated with the concept?

2. In what way has your concept "school" altered since you left school?

3. In the case of an orographical map, compare the concept of an average boy of ten with that of a trained geographer.

4. What kind of concept of "manhood" is a young child likely to have?

5. Take a poem, *e.g.* Wordsworth's "Nutting," or the first six stanzas of "Resolution and Independence," and describe (*a*) the images, (*b*) the meaning associated with the various concepts. In what respects would these be likely to differ in the case of the town and country child respectively?

SUGGESTIONS FOR READING.

J. Adams: *Exposition and Illustration in Teaching*. Ch. IV.

I. E. Miller: *The Psychology of Thinking*. Chs. XV.-XVII.

Thorndike: *The Principles of Teaching*. Ch. IV.

[1] Chamberlain, *The Child*, pp. 146-147. Such definitions are given by young children.

CHAPTER IV.

DEFINITION AND CLASSIFICATION.

1. The statements we make about any object fall into two main classes. A book is red, or large, or open, has leaves, etc. : it is on a shelf, or on the table, etc. The first set of predicates are its properties or qualities : the second set show its relation to other objects. It is not easy to distinguish between them : *e.g.* " This book is larger than that," seems to express both a relation between the two books, and a property of the first book. So with " *A* is the father of *B*." Indeed we often do express the properties a thing has, by comparing it with (or relating it to) other objects. All measuring or weighing does this.

In the present discussion we have to do with the properties of objects, whether they are made known by means of relations or not.

2. Every class name—horse, house, government, proposition—applies to certain objects, and it applies to these objects because they possess certain properties. Every adjective can serve as a class name : thus red, round, etc., apply to all objects which have these qualities. So also in general can every word

or combination of words which can serve either as the subject or the predicate of a proposition. But it will be convenient here to confine our attention to names which are used primarily as names of objects.

Children are occupied from their earliest years in learning the names of objects. "What is that?" is one of their most frequent questions. Thus a name with them refers primarily to things. And it is extremely important for children to have as wide and accurate knowledge as possible of the different things in the world around them, and of their names. When they come to the teacher they have a great amount of this knowledge; and one of the chief functions of education is to deepen and systematise it. One of the main methods of attaining this is to attempt to give accurate definitions: of a season, a ship, an island, a zone, a noun, a triangle, etc.

3. Let us imagine that we have collected together all the objects to which a name applies. The objects called by the name "box" would be very miscellaneous, large and small, wooden, iron, cardboard, etc. "Wood" will denote very many things, made of oak, ash, beech, birch, mahogany, walnut, etc. The name "ship" will denote objects such as sailing-ships, paddle and screw steamers. Notice the word "denote." The collection of objects called or denoted by a name is called the *denotation* of the name.

It is clear that these objects are not called by one name through mere accident. It is because they have certain qualities in common. A name does not merely denote certain objects: it denotes certain

objects which have in common the qualities which the name stands for. And when we are defining a name, it is these qualities on which our attention is chiefly fixed. Notice that we are not engaged in giving names to things: we are endeavouring to find out just what are the uses of the names at present given to things. We are endeavouring to clarify the vague concepts we have gained in our ordinary experience, and to make them logical concepts.

4. We must somehow compare and classify the properties possessed by the various objects in the class. Take for example the name "man." (Our analysis will hold good of any other name.) The properties fall into three groups.

(*a*) Those which all men have, and which no other objects have (*i.e.* which only men have). *E.g.* Capable of laughter, capable of using tools.

(*b*) Those which all men have, but which are possessed also by things which are not men. *E.g.* Having a body of a certain structure.

(*c*) Those which only some men have.

Clearly any property in (*a*) would serve to mark off the object from all other objects. But in general such a property is not enough for our purposes. We feel somehow that a great many other characteristics are significant in telling us what the object is. Indeed we feel that every one of the properties which all men have in common, must have close relation to the nature of man. Only men are rational and

capable of using tools; but all men have many other characteristics. If we omit all reference to these other characteristics, we shall miss a great part of the meaning of man.

5. Are we then simply to collect together all the properties in both (*a*) and (*b*), and call that the definition of man? This would give us too clumsy a definition, and in addition, does not bring us any nearer to understanding the nature of man. We want to make a *selection.*

There is a very important feature of all thought which will help us to see how this selection is to be made. It is that facts group themselves round one central fact. I may be puzzled to account for certain sounds until I discover by what instrument they are made. I may be puzzled to account for the movements of a particular person until I hear that he is a detective investigating a crime. In general the various things we see about us in ordinary life can be summed up in short phrases. What might seem to a savage the curious sight of a grimy man uttering loud cries, and standing on a cart containing strange black bags, is interpreted by us at once as a "coal-man selling coal." It is indeed this property of facts which enables us to have concepts.

In the same way the various qualities of an object throw light on one another. Man can count, use tools, laugh, is responsible for his actions. All these properties seem to be closely connected with his rationality. Again a triangle has its angles equal to two right angles, has any two sides together greater than the third, etc. All these are explicable as consequences of the fact that the sides are three in

number, and are straight. We have now a means of dealing with the manifold and complex qualities of an object. Careful and diligent inquiry will allow us to lessen the number by grouping them round a few significant qualities. This is a difficult process, and the results are partly recorded in language. The child learns the results while he is learning to speak; hence his concepts are affected by the work of generations of scientists.

6. Suppose we have grouped the multitude of qualities round a few prominent and central ones. These will form our definition of the object. Some of them will belong only to the object, while others will belong also to certain other objects. In the case of a ship, some of the qualities will belong only to ships, while others will be common also to all other vessels. A ship is a particular kind of vessel. Man is a particular kind of animal.

Thus, by its possession of qualities in common with other objects, the object we are defining falls into a class wider than itself. It is distinguished from all the objects in this wider class by possessing certain qualities peculiar to itself.

In this way, definition results in *Classification.* This process may be carried further. A triangle is a particular closed plane figure with straight sides. This is the class into which it falls. But this class falls into the wider class of closed plane figures; and this again into the class of plane figures in general, which falls finally into the class of figures. It is clear that language has already performed this classification for us. We see this classificatory habit in various modern words: steam-ship, air-ship;

mono-plane, bi-plane ; in phrases such as "The British Association for the Advancement of Science," etc.

7. Knowing all this, we can, in endeavouring to define a thing, shorten our work by asking, "Into what wider class does it fall ?" Thus, *e.g.* we could say at once that a ship is a vessel. The next question would be "In what way does it differ from the other objects in that class ?" A ship is a floating vessel. The test whether this definition was sufficient would be obtained by asking finally, "Are ships the only floating vessels ?" If we decide that they are, then we can accept the definition as satisfactory.

8. Our purpose, we said, was to systematise our knowledge of objects. There is still one set of properties which are important, and which we have so far neglected, namely, those in (*c*) : which are possessed by only some of the objects denoted by the name. An examination of the most important of these properties will enable us to discover sub-classes, and so give us detailed knowledge. Otherwise, our knowledge will remain vague and general. ***E.g.*** some ships are propelled by sails, some by paddles, some by screws : some are broad, some narrow. Our aim should be as complete an understanding as possible of the significance of these differences : *e.g.* in the case of ships, of the different uses to which the various kinds are put ; in the case of birds, of the different habits of life to which their differences in structure give rise, etc. This discussion of the sub-divisions which fall under any class is called *Division*. The student should work out for himself the conditions to be followed in sub-division,

if the division is to be made at once, (*a*) exhaustive, and (*b*) without cross-classification.

The wider class into which the objects fall is sometimes called the "*genus*": the various sub-classes are called "*species*": the characteristics which mark off any species from the main class are called the "*differentia*" of the species. Thus included in the genus "building" are the various species or sub-classes, house, church, barn, etc.; the differentia of the sub-class "house" is "used primarily as a dwelling place" (a house is a building used, etc.). Again, with respect to its various kinds, "house" could be regarded as the genus, being sub-divided into cottage, villa, etc., each with its own differentia.

9. In all this we have been passing from a knowledge of the actual objects to which a name refers (the denotation of the name) to a systematic knowledge of the properties in virtue of which the objects are called by the name. These properties are called the *Connotation* of the name. The words connotation and denotation are strictly correlative. The denotation of a name is the class of objects possessing the qualities included in the connotation of the name: and the connotation is the set of qualities possessed by all the objects included in the denotation.

If we think for a moment of any word: *e.g.* "triangle," we shall see that the number of qualities associated with it varies enormously from person to person. The geometry books define it as "a plane figure bounded by three straight lines"; we may know nothing more than that. To a beginner, it has certain other properties, some very perplexing; to the student, it has more properties, some obvious,

some wonderful, and many obscure. But the triangle itself has more than are known to anyone. How many of these properties are we to understand by the word connotation ? It has been found convenient to give different names (*a*) to the qualities commonly included in the definition of the name, (*b*) to the qualities known to any particular person, (*c*) to the sum total of those possessed by the object. The student should endeavour to find names which would be at once appropriate and concise for these three.

As a matter of practical importance, it should be noticed that misunderstandings in disputes may and often do arise through different persons associating different qualities with the same name : and the teacher should be particularly careful to remember how limited the child's knowledge is. In teaching we have to pass from the qualities known to the child (*b*), to an account of the qualities the object itself possesses (*c*), systematised and rendered precise by means of the definition (*a*).

10. A good definition, as we have seen, should be one which throws the greatest possible amount of light on the properties of the object. This, however, is in many cases difficult, and only to be obtained as the result of careful investigation into the inner structure and nature of the object. Good definitions are indeed the result of science, and depend on scientific hypothesis. In many cases we are compelled to accept provisionally a definition which merely serves to mark off the thing defined from everything else. Definitions which simply do this are called Descriptions or *Descriptive Definitions.*

We have them in all the biological sciences. While the properties included in such a definition do not throw light on the other properties, yet they serve as an index or mark of those properties. Such definitions are for temporary use ; and as our knowledge grows, our definitions become more and more perfect. Flowers are distinguished chiefly by means of their shape, and the position of their parts. At one time there seemed to be very little connection between the shape of a flower and its other characteristics. The theory of evolution has thrown great light on this connection in many cases, and where this is so, the definitions become more than mere descriptions. But our knowledge is still very imperfect ; and the definitions remain, for the most part, merely descriptive.

11. There are certain conditions, the reason for which will now be evident, which can and must be satisfied by every definition.

(*a*) We have seen that it should serve to mark off the term defined from everything else, and that it should contain only characteristics which are possessed by all objects denoted by the term.

Thus if an island is defined as " a piece of land entirely surrounded by water," the definition is faulty unless (1) All islands are so, and (2) All such pieces of land are islands. In general, if *A* is defined as *abc*, the definition is faulty unless All *A* is *abc* and All *abc* is *A*.

(*b*) In the second place, we saw that in our definition we wanted only the central properties. A definition is faulty if it contains more properties than are necessary. *E.g.* " A square is a plane figure

bounded by four equal straight lines, with all its angles right angles."

(*c*) The properties must be such as really do throw light on the term to be defined. There are several things to avoid in order to satisfy this condition.

We must not define a term by properties which can only be understood when we understand the term we are defining. Such a definition would imply the term defined, and would be called *circular*. Take the following from "Punch":

Tourist (pointing to molehills): "What are these?"

Giles: "Them's oompty-toompties."

Tourist: "And what are umpty-tumpties?"

Giles: "Oompties wot the toompties makes."

Tourist: "But what are tumpties?"

Giles: "Whoy, wot makes the oompties, you fool."

Here the explanation makes a full circle. Again, "Fear is what you feel when you are afraid to face danger." "A swing is what you swing on. Swinging is what you do on a swing." We are tempted to rest in such definitions whenever we are dealing with something with which we are perfectly familiar, but which is not easily classified.

The properties must be more general than the term itself. When we are endeavouring to explain a new word or a new object to a child, we must use only words which are familiar to him. It is of no use to explain a chronometer as a mechanical instrument for the precise measurement of time; or a bishop as a high dignitary of the church. But from a scientific point of view, these could be regarded as good definitions.

They must not be mere metaphors. A metaphorical description is not useless, but it is not a definition. It may suggest much for our consideration, as a result of which we may, by our own activity, arrive at a correct definition. *E.g.* "A discontented man is one that is fallen out with the world, and will be revenged on himself. Fortune has denied him something, and he now takes pet, and will be miserable in spite." (Earle). Here many things are presented for our thought, and we feel we understand better the reasons why a man is discontented: but this is not a definition. It may lead to a definition.

The properties must not be merely negative. "Sleep is what you do when you are not awake." But often a definition which is apparently negative is really not so. For instance, lines are either straight or curved: a straight line is one which does not change its direction (apparently a negative definition, but really amounting to "one which keeps the same direction"). A curved line is one which is not straight (apparently negative, but really amounting to "one which changes its direction"). Hence in some cases it is a matter of convenience whether the form is positive or negative. In some cases it seems difficult to give anything but a negative definition. Darkness is the absence of light. A bachelor is a man who is not married. What is desired is, that the definition should always refer to definite positive characteristics, whether its form be negative or not: and it is advisable to make the definition affirmative wherever possible.

These three conditions may be called those of (*a*) accuracy, (*b*) precision, (*c*) clearness.

Accuracy is satisfied when the definition marks off the thing defined from all other things, and refers to all the things defined. Precision is attained when this marking off is done by means of the fewest and most central characteristics possible : and clearness may be regarded as a result of these two conditions.

EXERCISES.

1. Divide into three groups (as in Section 4) the properties possessed by (*a*) horses, (*b*) watches.

2. Give a descriptive definition of Glasgow, the sun, a motor-car. Could a scientific definition be given of any of these ?

3. What is (*a*) the denotation, (*b*) the definition, of man, penny, railway-track, king ?

4. The three kinds of connotation mentioned in Section 9 are ordinarily called (*a*) conventional, (*b*) subjective, (*c*) objective connotation. Illustrate in the case of the objects mentioned in Questions 1, 2, 3.

5. Do the definitions you have already given bear out the statement that in most of our definitions we classify ?

6. What questions would you ask in trying to define a word ? Is there any shorter method than that of merely collecting all the properties and then dividing them into groups ?

7. " A term can be defined by giving its genus and differentia." Explain.

8. Are the following definitions good ?

A king is a constitutional ruler.
A circle is the locus of points equidistant from a point.

Parliament is a body of men chosen by the people to make laws.

A brake is a mechanical device for reducing motion by means of friction.

History is an account of events which happened in the past.

A nail is a piece of iron with a head, to drive through things and keep them together.

A pentagon is a figure with five equal sides.

Stupidity is slowness of mind in word and deed.

A rational integral function of x is one in which there are no powers but integral ones, and where the coefficients of the various powers do not involve x. Is this a negative definition ?

9. What are the virtues and defects of the following if regarded as definitions ?

Revenge is a kind of wild justice ; which the more man's nature runs to, the more ought law to weed it out. (Bacon.)

A plodding student has a strange forced appetite to learning, and to achieve it brings nothing but patience and a body. (Earle.)

10. Discuss the following definitions :

The subject of the verb is what is spoken about.

A verb is a doing word.

A relative pronoun is a word that refers to some noun going before, called its antecedent.

A phrase is a group of words.

11. Discuss the following :

Heat is a mode of motion.

A fortress is a stronghold.

A liquid is anything that runs.

The climate of a country is its average weather.

12. Make a sub-division of closed plane figures, so as to include all the figures dealt with in ordinary geometry, dividing according to number and straightness of sides, and according as none, all, or some of the sides are equal.

13. Is the following division accurate ?

Ships are propelled either by steam or by sail ; some are broad, some long, some flat-bottomed ; some are barques, some cutters, some yachts.

14. Suggest ways in which children might be led to give the following definitions :

The subject of a sentence is the first noun.
A participle is a part of a verb ending in "ing."
The infinitive is the part of the verb with "to" before it.
Intransitive verbs do not pass over.
Poetry is writing which rhymes.
A cuckoo is a bird that does not lay its own eggs.
A challenge is a question with force.

15. Distinguish between division in the logical sense, and in the sense indicated by such a sentence as "Great Britain can be divided into England, Wales and Scotland."

16. What are (*a*) synonyms, (*b*) homonyms ?

CHAPTER V.

DEDUCTIVE INFERENCE.

CONCEPTS are, of course, for use. It was simply because we desired to make as full and accurate use of them as possible that we found it important to render them precise by definition and classification. In this chapter we are to study more particularly one aspect of the uses to which we put concepts, being already familiar with those uses in thought and action.

In discovering what a thing is, we have to find some character in it which enables us to place it. We see an object in the distance. Its general size and shape put certain possibilities out of the question. It may be a horse, or a man, or a bush, but not a tower or a mountain. If it is a man, we shall see him move. Here we are appealing to our knowledge of both men and bushes; men move from place to place, bushes do not. It moves. But this does not tell us with certainty that it is a man. How shall we distinguish a man from other animals? Our knowledge replies, By shape. Hence we go nearer.... It turns out to be a man.

Now we have argued thus with ourselves at the various points :

Clue : It may be an animal or a tree.
Test : Does it move ?

No trees move.
This moves.
∴ This is not a tree.
All men move.
This moves.
∴ This may be a man.

At this point we could say, "This is a man," only if we knew that whatever moves is a man. We do not know this, and hence we had to apply a further test :

All things with a particular shape are men.
This has this particular shape.
∴ This is a man.

This passing from a few properties of an object to the nature of the object is called *Inference.* That aspect of it in which we make use of our previous knowledge to suggest clues, and in which, when we have decided on the most probable, we use our previous knowledge to suggest tests, is called *Deductive Inference.*

We employ the same process in endeavouring to solve a problem in connection with an object which is already familiar to us. We have to think of the object in a new way ; in terms of something which shall throw light on our problem. This is well illustrated by the following : "Can you farm profitably on the banks of the Nile ? Think of the Nile as a river with such and such a river basin and as a

river with an annual overflow. How shall you bridge the Nile ? Think of the Nile as so wide and deep and with such and such a bottom. How far can you sail up the Nile ? Think of the Nile as so deep, with such and such falls and cataracts. Shall a town pump its drinking water from the Nile ? Think of the Nile above the town only and of its sources of contamination." [1]

The same holds in solving problems, *e.g.* in Arithmetic. The student's recurring question is : What rule do I use ? He should be led to recognise the discovery of the rule as the problem and hence as something which ought not to be given to him. Generally, examples are arranged under "heads," so that there is no doubt as to the rule : these should be recognised as merely preliminary examples, to afford the student practice in seeing the rule exemplified in various cases ; and the importance should be emphasised, of miscellaneous examples, where the student is thrown back upon himself.

The bringing to bear of a general rule on a particular case is, as we have said, deduction. We have proved that the object we saw in the distance was a man, by means of deductive inference, by bringing the particular case before us under a general rule. We prove a proposition in geometry by the application of general rules. Here, then, is one of our instruments of proof : deductive inference. Once we are in possession of general rules (What moves is either man or animal ; When the signal is down the train will shortly come ; When the clock strikes one it is the half-hour) we can prove certain results by

[1] Thorndike, *Principles of Teaching*, p. 152 (cf. pp. 160-164).

the application of these to our particular situations. We are satisfied when we can do this. Proof seems complete. Now as we saw, the central problem for Logic is, "What constitutes proof?" Deductive inference is one of the instruments of proof, once we are in possession of general rules. Hence, if we inquire, under what conditions can we make a deductive inference—when can we apply a general rule to a particular case?—we shall have answered one of the questions Logic has to raise. (We shall still be left with the problem, as to where the general rules come from: and we shall have to answer that later.)

This study is known as Formal Logic: it will lead us to examine more carefully the nature of thought. We shall find that thought itself has a definite nature, which is expressed in those principles which we apply whenever we think. They are principles which are common to all the sciences. For, since all the sciences are the result of thought about the properties of objects, we should expect them all to express the nature of thought. Just as a great writer impresses his own mind on all his books—or a great burglar his own character on all his burglaries—so thought stamps with its own impress all the products of its own activity. If then we were to examine all the principles involved in the various sciences, and disentangle those principles, which, though appearing in different dress, are yet essentially the same, we should have brought to light the principles which belong to thought itself. We have already become familiar with some of them in their application: we have now to consider

them in themselves. For it is only by considering the principles of thought that we can decide under what conditions we can make a deductive inference.

EXERCISES.

1. What kind of consideration must be thought of in answering the following questions in Geography :

   Why is the East Coast of England drier than the West ?

   Why is the china-clay of Cornwall sent to the Potteries, and not manufactured into porcelain on the spot ?

   Why is cotton grown on the Deccan ?

   Why are there no large inland towns in Ireland, such as Birmingham in England, or Motherwell in Scotland ?

   Why are the rivers of Spain of little use for navigation ? (Mort, *Regional Geography*.)

Note the processes in your mind in answering these questions.

2. Discuss the following and the light it throws on the use of concepts :

   " I noticed in the farm-yard a large pile of small wooden trays, stacked and thatched over like a hay-rick, and I could not imagine what they were for. A day or so later I noticed a small pile of the same trays in a potato field. I immediately concluded that they had something to do with potatoes. At first I thought that they were for carrying the potatoes : such being my ignorance of potato-digging. On inquiry, I found that they were used for storing the potato-seeds during the winter."

3. Show how all exercises in "parsing" are exercises in deductive inference.

4. Discuss the following, and the light it throws on the search for a cause:

> Swansea smelts copper, which it used to get from Devon. Why was the copper not smelted in Devon? *Ans.* Because there is no coal in Devon. *Q.* But why not bring coal from Swansea? *Ans.* Because it is cheaper to take the ore to Swansea. *Q.* Why? *Ans.* Because coal is heavier. Not so: bulk for bulk it is lighter.

SUGGESTIONS FOR READING.

Bagley: *The Educative Process.* Ch. XX.

I. E. Miller: *The Psychology of Thinking.* pp. 251-260.

J. R. Seeley: *The Expansion of England.* Lecture VIII.

Lewis Carroll: *Alice in Wonderland.*

CHAPTER VI.

THE FORMAL ASPECT OF REASONING.

1. Before we proceed there is a point which must be clearly understood. It may perhaps be approached by bringing out what is meant by the word "formula" in Mathematics.

There is a general formula which enables us to square *any* number. Examples will indicate what it is:

$$68^2 = 70 \times 66 + 4.$$
$$83^2 = 86 \times 80 + 9.$$

To square 84. What is the nearest ten? 80. This is got by subtracting 4. When we *add* 4 we get 88. The square is

$$88 \times 80 + 16.$$

To square 87. What is the nearest ten? 90. This is got by adding 3. When we subtract 3 we get 84. The square is

$$90 \times 84 + 9.$$

Thus *e.g.*

$$68^2 = (68 + 2)\ (68 - 2) + 4.$$
$$65^2 = (65 + 5)\ (65 - 5) + 25.$$

The general formula is,

$$a^2 = (a+b)\ (a-b) + b^2,$$

which applies to all numbers, and is a simple consequence of the identity,

$$a^2 - b^2 = (a+b)\ (a-b).$$

2. We see here the value of such a general rule or formula. It throws light on what at first might appear to be very mysterious and very difficult to prove. It does not limit us as a particular rule would. Hence we understand the reason for what we are doing, and in addition we are able to deal confidently with all cases. A general formula of this kind aids both theory and practice.

What have we done to all the numbers above treated ? We have thrown them all into the same *form*. It was only because they could be thrown into the same form that they could be so treated. We prove this by using a symbol which stands for any square. The expressions in Algebra are then general. Algebra thus lays stress on the forms into which numbers can be put. In this sense, Algebra is a *formal* science.

3. Let us look at propositions. Take the following examples :

If a horse is four-footed, then four-footed things are sometimes horses.

If all men are mortal, then some mortals are men.

If children are always inquisitive, then an inquisitive being is sometimes a child.

All these can be brought under the same form, that of the second : which we can express generally,

If All S is P, then Some P is S.

Thus wherever we have a proposition which can be thrown into the form "All *S* is *P*," we can deduce a proposition which can be thrown into the form, "Some *P* is *S*."

Propositions, then, can take general forms, just as numbers can. If we can discover just what forms propositions can take, and what consequences can be deduced wherever these forms appear, we shall have a science which deals with propositions in a manner similar to the way in which Algebra deals with number.

4. Take another example. If sugar is always sweet, and a particular thing in front of us is not sweet, then, however much it may resemble sugar in other respects, it is not sugar. If all books have leaves, then a thing which has no leaves is not a book. Both of these have the same form, namely, "If All *S* is *P*, and a particular thing is not *P*, then it is not *S*." And when we have dealt with this, we can deal at once with everything which can be thrown into this form. But is there any point in dealing with the general form? An example will perhaps help us to answer this question.

5. Suppose I hear a noise in the street, and say, "There is a motor-car passing." If you ask me how I know, I reply perhaps, "Because all motor-cars make a noise like that." Is this a sufficient reason? In the first place you may point out, that when I say, "All motor-cars make such a noise," I do not cut off the possibility that other things besides motor-cars do so also; and hence I have not finally proved that the thing in the street is a motor-car. It may be something else. In order to prove my point I

should have to say, "Only motor-cars make that noise."

But if you were not able to convince me by simply pointing this out, you would proceed to take other cases. You might take the simple case, "All horses are quadrupeds," and ask me if I think that everything with four legs is a horse ? You would go on, through various instances, until I was quite convinced, that when I say, "All motor-cars make that noise," I do not say, that whatever makes that noise is a motor-car.

6. Now what have a horse's four legs to do with the noise made by a motor-car, that your mentioning them should help me ? Clearly that was not what you wanted me to attend to. What you intended me to think of was, what we *mean* when we make a statement of the same *form* as "All horses are quadrupeds," "All motor-cars make that noise." In other words, you were really drawing my attention to the meaning of the form "All *S* is *P*." Your suggestion was that if my statement adequately expressed the reason in the case of the motor-car, then a similar statement ought to hold in the case of horses, and in general, wherever the same form is met with.

I should indeed finally be convinced that I had not said quite what I meant to say ; that what I meant was, that the noise I heard was one peculiar to motor-cars : *i.e.* that only motor-cars make that noise.

7. Can we state clearly what principle is involved here ? It is this : that all reasoning, all inference, when completely expressed, has a formal side in the

sense explained. If your reasons are good in this case, they must be good in any case which presents the same form. If not, they are not good in this case either. They are either actually wrong, or else you have not completely expressed all you mean.

E.g. "He is poor because he is honest." Here we recognise that honesty is not the complete reason for his poverty; for if it were so in this case, it would have to be so in all cases: and we do not intend to assert that all honest men are poor. Hence something more is implied than is expressed. He is poor because, being in the circumstances he is in, or being the person he is, he is honest. We mean then, that any person of his nature, or in his circumstances, who was honest, would be poor. And we could go on to point out just what particular features in his nature, or in his circumstances, combined with his honesty, cause his poverty.

8. Thus in reasonings on various matters we see similar forms involved; and when we discuss a particular reasoning we often in effect simply disentangle the form. In *all* propositions of the same form as "All men are mortal," "All horses are quadrupeds," we have certain characteristics; *e.g.* it follows that "Some mortals are men," "Some quadrupeds are horses," etc. We shall understand these propositions better when we have succeeded in attending to the form which is expressed in all of them. The form of the proposition is thus a characteristic which we pick out for a certain purpose. We must then prevent irrelevant considerations from interfering with this. We have seen this process in the previous chapter. Suppose someone were to

ask us, " Will a white cat weighing 8 pounds counterbalance two weights each of 4 pounds ? Remember, it's a Persian cat." We should think of the introduction of the fact that the cat is Persian, and white, as silly ; since for our purpose, we want to know only its weight. For different purposes different things are relevant. And there is an aspect of all reasoning in which it is sufficient to attend to the *form* which the reasoning takes. Now the propositions " All men are mortal," " All horses are quadrupeds," etc., are all *of the same form as* " All S is P " (or briefly, *of the form* " All S is P "), where S and P stand for something intelligible whose nature we do not trouble to specify any further. S and P are symbols ; their use enables us to express with the maximum of clearness just what is the form embodied in such propositions as those given above, and to study it without being hampered by irrelevant considerations.

Formal Logic studies the form into which propositions and reasonings can be thrown. Accordingly in what follows, our attention will be occupied with symbolic propositions such as " All S is P," " All A is B," etc. The procedure of Formal Logic is in this respect similar to that of Algebra, which works with symbols which stand for numbers. If Algebra endeavoured to work with particular numbers, its operations would be encumbered with so much irrelevance that it would make very little progress. The reader will find it economical to acquire an early familiarity with the use of symbols in Formal Logic.

CHAPTER VII.

THE PREPARATION OF THE MATERIAL.

1. Whenever we recognise an object, we make a judgment. Whenever we look at an object in the light of a general principle, for the sake of solving some problem in connection with it, we make a judgment. Indeed all attempts to think about objects give rise to judgments. Concepts are formed, as we saw, in the process of making judgments, and testing them by experience. When we are dealing with things for which our concepts are already more or less adequate, the judgments we make can be described as concepts in use. In using a concept in this way we do in general modify it; and thus what from one point of view can be called the use of a concept can from another point of view be called the formation of a concept. In what follows we shall attend more to the former aspect. But the latter aspect must not be forgotten. When I say, then, "That is a dirty table-cloth," or "That table-cloth is dirty," I am using my concept "dirty table-cloth" as defining the important thing about the situation in which I am placed. When a burglar, trying to get into a house, notices a window unbolted

and says, "There is an unbolted window," or "That window is unbolted," he is defining, by means of the concept "unbolted window," that aspect of his situation which will enable him to proceed. When I say, "It is raining," "It is half-past eight," I am throwing light on my situation by means of the concepts involved, as centres from which to start in thinking or acting. Similarly, when the mathematician, after puzzling over a problem, suddenly discovers, "That is a right-angled triangle," or "That triangle is right-angled," he is defining the important aspect of his situation by means of the concept, "right-angled triangle." To define a situation by means of a concept is to make a judgment. It should be noticed that in our formulation of the judgment we may break up the concept: "unbolted window" is the aspect of his situation on which the burglar seizes, because of its value for him; he may say, "There is an unbolted window," or he may seize on one element within this, namely, "unbolted." It is the "unbolted window" which helps him out of his situation; it is the "unboltedness" of the window which makes the window useful. In this case, breaking up the concept he formulates the judgment, "That window is unbolted."

2. The material with which Logic deals is the judgment. What is the form any judgment takes? If there are various forms, what are they?

That is our first task. Perhaps the best way of setting about it would be to examine all the judgments which occur in ordinary speech and thought, and try to classify them.

In the first place we must ask, "Do all sentences express judgments?" Knowing what we do about judgments, we can answer, "Any sentence whose main object is to express a fact (or define a situation), expresses a judgment." *E.g.* a question indirectly conveys information, but that is not its main object. "Is it raining?" conveys to you the information that I desire to know whether it is raining or not; but its chief object is to elicit information. So with a command or a wish. Some interjections convey information, and this can be regarded as their main purpose, as "Fire!" "Thieves!" Others are more in the nature of commands, as "Look out!" "Help!"

A clear test as to whether a sentence expresses a judgment is: Is it capable of being true or false? If so, it expresses a judgment. Thus it would be improper to regard "Would I were a king" as either true or false. The characteristic thing about a judgment is that it claims to be true. Every judgment is an *assertion*.

3. Can we classify those sentences which express judgments, so as to bring out the various forms of the judgment? Let us take a few examples at random:

It rains.

The grass is green.

Some countries are prosperous.

The prime minister made an important announcement to-day.

It is expected that there will be a general break-up of the ice in a few days.

No one knows the precise details.

If he had known, he would have exercised more care.

France views with anxiety Germany's projected increase in her army.

Violets grow on that bank in March.

It is either all or nothing.

Capital and labour were bitterly opposed.

4. It is clear at any rate that there is an enormous variety in the manner in which judgments are expressed. The work of classifying them must have involved great labour on the part, not of one man, but of many; and we shall here simply accept the results of that labour.

Judgment falls into three main kinds: categorical, hypothetical, and disjunctive.

Categorical judgments assert a fact directly, hypothetical judgments assert a fact under a condition, while disjunctive judgments assert an alternative.

We shall discuss them in turn, confining our attention meanwhile to categorical judgments.

5. Categorical judgments are many and varied. We have seen that in all of them we are defining a given situation by means of a concept, and that in our formulation of the judgment we may break up the concept. In many of the categorical judgments we make, we analyse the concept into two elements. The grass is green. That window is unbolted. Thus many of them take a form in which something is asserted or denied of something else. In a large class, however, there is no explicit "subject" at all. It rains. It is warm. In others there does not seem to be one subject alone, but several. Capital and labour are opposed. This is not merely, Capital

is opposed, and labour is opposed. It is the opposition of capital and labour as defining the important point in a given situation. To analyse the concept by means of propositions each containing two elements would involve, Capital is opposed to labour, and Labour is opposed to capital, asserted in one sentence. Similarly in, Violets grow on that bank in March, there does not seem to be any one subject definite. The concept is the growing of violets on that bank in March : and if we wanted to express it by means of sentences with one subject and one predicate, we should have to interpret it as asserting the growing of violets in March, as a thing taking place on that bank ; the growing of violets on that bank, as an event happening in March ; violets, as the flowers growing on that bank in March, and so on, combined in one sentence.

6. Originally, for the sake of simplicity, logicians put all these categorical propositions into one form : that in which we have something affirmed or denied of something else. It was found that for the ordinary purposes of inference, categorical propositions could in general be treated as if they did assert or deny something of something else. It was indeed thought that this was the only form of categorical proposition with which Logic was competent to deal. To put a proposition into this form was called putting it into "logical form."

Thus in every categorical proposition, when put into logical form, there is a subject (that spoken about) and a predicate (what is affirmed or denied of the subject). The subject may be a single object (as in "The door is shut"), or a whole group of

objects ("All Europeans are white"), or a part of a group ("Some countries are prosperous"). We have then six forms, made use of by Formal Logic. We can symbolise them as follows :

Affirmative : S is P.	Negative : S is not P.
All S is P.	No S is P.
Some S is P.	Some S is not P.

Notice the form all these propositions take. In all there are three parts, Subject (symbolised by S), Predicate (symbolised by P), and Copula, *is* or *is not*.

At present we draw attention to the last point—the copula *must*, for the sake of logical form, be part of the verb "to be." Thus *e.g.* in "All trees have leaves," the copula must be made explicit : "All trees are things having leaves."

It is extremely important to be quite precise in this matter.

Exercises.

1. Give circumstances in which the following judgments would be significant :

The window is broken.
The room is small.
The ball is round.

State the concepts, and indicate in what way they would serve as starting points for thought or action in the circumstances you have given. What is the relation of the predicate to the whole concept ?

2. What are the concepts in the following :

There is a full moon to-night.
It is half-past eight.
The general praised the soldiers for their bravery.
It is better to take the long road through the valley than the short one over the hills.

Treat these propositions as in Question 1.

7. Certain explanations remain to be made.

(*a*) S is P. S is not P.

These are called singular propositions, because the subject is a single thing.

The President of the United States is a Democrat.

This chair is broken.

Notice that certain propositions seem to be singular which are really not—*i.e.* they are singular in form but not in meaning. *E.g.* "The President of the United States is chosen by the people." This is said not only of the present President but of all Presidents. Its form must then be altered, so as to bring out its true meaning: "All Presidents of the United States are chosen by the people," where form and meaning are in agreement.

A proposition is a singular proposition if its subject can refer to one, and only one, individual. Otherwise it is not, and its form must be altered.

Exercises.

State which of the following are singular propositions:

A triangle cannot have all its angles obtuse.

A triangle is drawn on the diagonal of a square.

An island is a body of land entirely surrounded by water.

An island is situated at the mouth of the river.

The dog barked excitedly.

The sheep and the cow have no cutting teeth.

The teller of tales is of necessity a monopolist.

A river must not be thought of merely as a stream of water.

A man that is young in years may be old in hours, if he have lost no time.

He that hath wife and children hath given hostages to fortune.

8. (*b*) All *S* is *P*. No *S* is *P*. *S* is here a class name.

All Europeans are white.

No rivers are salt.

These propositions are called universal : affirmative and negative. In these the predicate is affirmed or denied of every member of the class *S*. A class name may be used either distributively or collectively. When a predicate is affirmed or denied of every member of the class denoted by the subject of a universal proposition, then the class-name is used distributively. When the predicate is affirmed or denied not of every member, but of the class as a whole, the class-name is used collectively. *E.g.* "All the soldiers were disembarked in five boats," "All the soldiers were supplied with a new rifle." Now in Logic class-names are always used distributively. Hence in the case of the sentence, "All the soldiers were disembarked in five boats," we shall have to make some alteration of the form ; since this, if it appeared as a logical sentence, would have to mean, "Each soldier was disembarked in five boats." *E.g.* It will be correct to say, "The whole army was disembarked in five boats"—a singular proposition.

In "All *S* is *P*," "All *S*" is the range over which the predicate *P* is asserted. If "All *S*" includes S_1, S_2, S_3 . . . S_n, then the proposition asserts that "S_1 is *P*," "S_2 is *P*," . . . "S_n is *P*."

In "No *S* is *P*," the predicate *P* is denied of every member in *S*. The proposition asserts "S_1 is not *P*," "S_2 is not *P*," . . . "S_n is not *P*." It would be conducive to symmetry if we could have written

this simply "All S is not P." But this form in ordinary speech does not mean "No S is P." "All Englishmen are not patriots" means "Not all Englishmen are patriots"; *i.e.* though many are, yet some are not. The form "No S is P" is more natural: but we should always think of it, for the purposes of Formal Logic, as meaning "S_k is not P" asserted of every member in S.

Exercises.

1. Put the following propositions into logical form:

   All the angles of a triangle are equal to two right angles.
   But a triangle has three angles.
   Hence a triangle has altogether six right angles.

2. Point out which of the following propositions are true universals, and put them into logical form:

   All roads lead to London.
   All roads don't lead to London.
   Each man prepared to depart.
   Any book which fell into his hands was eagerly read.
   All that glitters is not gold.

3. Put into logical form all the propositions given in the exercises in Section 7 which are not singular.

9. (*c*) Some S is P. Some S is not P.

   Some countries are civilised.
   Some men are not industrious.

These propositions are called particular: affirmative and negative.

There is one thing which will strike the student as odd: and that is, the meaning of "some." When

we say ordinarily, "Some countries are civilised," we mean, some are, and some are not. In other words, we mean, some only, or some not all. And when we say ordinarily, "Some men are not industrious," we mean, some are and some are not. But if so, then there would seem to be no difference in meaning between "Some *S* is *P*" and "Some *S* is not *P*."

Hence in Logic, "Some *S* is *P*" means, "Some *S* at least is *P*," leaving open the question as to whether "All *S* is *P*." Similarly, "Some *S* is not *P*" means, "Some *S* at least is not *P*."

"Most *S* is *P*," "Few *S* is *P*," "A few *S* is *P*," etc., fall under the head of particular propositions. "Most *S*" means, "the majority of *S*"; we shall here, for simplicity, represent it by "Some *S*." "Few *S* is *P*" means, "Most *S* is not *P*, and the minority of *S* is *P*"; we shall treat it as asserting the two propositions, "Some *S* is *P*," "Some *S* is not *P*." What will be our treatment of "A few *S* is *P*"?

Exercises.

Put into logical form the following sentences:

> Most of the women and children were saved: few of the men. A few of the boats had been provisioned; but almost all were without water. An experience such as this is unforgettable: not all the description in the world can convey an adequate idea of it. A man lives through horrors intensified by the suffering all round him.

10. There are certain forms we shall often meet with, which must be put into logical form.

(*a*) "Only S is P." "Only Irishmen are in that regiment." This = "None but Irishmen are in that regiment" = "No persons who are not Irish are in that regiment." Thus "Only S is P" = "None but S is P," and is represented in logical form by "Nothing which is not S is P," or, more shortly, "No not-S is P." (= No K is P, where K = not-S.).

(*b*) "S is sometimes P." "S is not always P." "S is generally P," etc. The question to ask always is, How much of S is P? Thus the first of these says that "Some S is P." The second that "Some S is not P." Notice how rigorously we keep to our six forms.

(*c*) "Wherever you have S you have P." "You can only have S in the absence of P." "If S is absent, P is generally found." These can be written, "The presence of S always means the presence of P." "The presence of S always means the absence of P." "The absence of S sometimes means the presence of P." With these hints, it will be seen that they can be represented by the forms, "All (S) is (P)," "No (S) is (P)," "Some (not-S) is (P)": where (S) means, "cases of the presence of S," and (not-S) means, "cases of the absence of S."

Other forms will be met with, in which the student must use his own ingenuity.

11. All this is a preparation of the material for the sake of simplification. We had in the first place carefully to select certain forms as our standard forms, and to assign to each of these forms one, and only one, meaning. We had then to throw the judgments of ordinary life into one or other of these forms. Sentences which express judgments in

ordinary life are our specimens, but they have to be prepared—cut and dried—before we can use them. We are doing with our material just what the botanist or zoologist or geologist does with his. Or, to take another illustration, we are sawing our trees into planks of certain forms, in order the more readily to get to work. In putting propositions into logical form we do not get all the meaning of the proposition : nor is the form always natural : but we can, and must always see that we do, get enough of the meaning for our purposes.

The subject and predicate of a logical proposition are called *terms*.

12. What we have said is sufficient to indicate that Formal Logic does not claim to discuss the whole nature of reasoning. We do not pretend that our " cut and dried " forms in any way adequately represent the living processes of thought ; but we do claim that there is an aspect of complete reasoning which can be adequately treated by Formal Logic. We shall meet this limitation in another form in Ch. IX., § 15.

EXERCISES.

1. Put into logical form the following :

Unmarried men are best friends, best masters, best servants, but not always best subjects, for they are light to run away, and almost all fugitives are of that condition. (Bacon.)

A knight seldom stirred from his house without a falcon on his wrist. In the monuments of

those who died anywhere but on the field of battle, it is usual to find the greyhound lying at their feet, or the bird upon their wrists. Nor are the tombs of ladies without their falcon. (Hallam.)

Peat bogs are found in the north and west of Jutland.

The Rhine is the only river which flows across both South and North Germany.

The forests of the Scandinavian peninsula consist of firs and pines. Their tall straight stems make masts for ships, or are sawn into planks to build their keels and decks. The houses in the forest area are generally of wood, often beautifully carved and painted. The trees are felled in winter and dragged over the frozen ground to the rivers, down which they float when the frost breaks up in the spring. (Herbertson: *Junior Geography.*)

The orang is sluggish, exhibiting none of that marvellous activity characteristic of the Gibbons. Hunger alone seems to stir him to exertion.

Most can raise the flower,
For all have got the seed.

Just nineteen politicians are Cabinet Ministers.
A Cabinet Minister is human.
Hence, just nineteen politicians are human.

2. From the definitions of a singular proposition (p. 51), and of a term (p. 56) collect the definition of a singular term.

CHAPTER VIII.

THE LAWS OF THOUGHT.

1. Certain principles which arise out of the nature of the singular proposition are called Laws of Thought. They were once supposed to be the only laws of thought, but now it is recognised that this is not the case. They express the most elementary and fundamental characteristics of the nature of predication in its formal aspect.

2. (*a*) Every object has many properties. In the case of a particular object we know some of them, and are ignorant of others. Certain properties are so connected that where one is, the others will be. We may know this, without knowing whether either of them belongs to a particular object. But if we discover that an object has the one property, then we can say without further examination, that it will have the others. This indeed is, as we saw, what makes it possible for us to form and use concepts.

If now we think of the *judgments* we make about objects, what we have already said comes to this : every subject can have many predicates, and we may know some of the predicates and be ignorant of others. We may know that certain predicates are

always connected, without knowing whether any of them apply to a particular subject. But if we discover that one of them does apply to the subject, then we can say, without further examination, that the others will. This is the simplest case of deductive thinking.

Thus it is clear that the most elementary statement we can make about predication is :

Every subject can have many predicates.

2. (*b*) Every singular term is predicable of itself.

This house is this house.

The Prime Minister is the Prime Minister.

The general formula is "*A* is *A*."

This may be called the Principle of the Identity of Terms.

This may not seem of much importance, but it is like the law of the land : negligible so long as we do not break it. If we use a term in two places in such a way that the term as it appears in one place cannot be predicated of the term as it appears in the other, then our reasoning cannot be correct. In other words, a term must not be used in two incompatible senses.

Ambiguity of words makes this very difficult to avoid. We shall meet it later. The only way to avoid it in long or complicated discussions is to fix clearly the meaning of all important terms at the very beginning, and to keep strictly to the meanings throughout the whole discussion. Technical terms are the result of this. In certain subjects—Political Economy, Art, Literature—we find long discussions as to the various shades of meaning a term possesses, which may seem at first sight unnecessary, but which are really of great importance.

3. (*c*) If S be any singular term, and P be any predicate, then the two propositions "S is P," "S is not P," cannot both be true. The one contradicts the other. One must be false. In ordinary life the law appears in the form of the maxim: "You must not contradict yourself." But for the sake of accuracy it is imperative to state the law in the above form. It is called the Law of Non-contradiction.

This horse is brown.	This horse is not brown.
This plate is made of gold.	This plate is not made of gold.
England is a geographical term.	England is not a geographical term.

Consider these three pairs. The Law says that both statements cannot be true.

It may be objected that the horse is partly brown and partly not, and hence both statements are true; or that looked at in one light it is brown, and in another light it is not.

Similarly it might be objected that one side of the plate is gold and the other side is silver: or that the plate is made of a mixture of gold and silver; and that hence both statements are true.

In the third case it might be said that clearly England can be considered as a geographical term, or again as a term which signifies a people under a certain form of government, with a common national life, etc., and that thus both statements are true.

But a closer consideration will show the mistake. We have not been sufficiently in earnest with the law. The subject is to remain the same, and the

predicate is to remain the same. And in order to be certain that we are keeping the terms the same, we must make them perfectly definite.

E.g. The whole horse is brown (seen from a particular point of view).

The whole horse is not brown (seen from this same point of view);

or again, A definite part of the horse is brown.

The same part of the horse is not brown.

Thus we must keep the same point of view, and speak about the same thing (or the same part): the application of the law is then clear.

In the third case, we are clearly trying to give two meanings to the word England.

4. (*d*) If S be any singular term, and P any predicate, then of the two statements "S is P," "S is not P," one must be true. Both cannot be false. There is no middle course between affirming and denying P of S. If you refuse to affirm that S is P, you are thereby compelled to admit that S is not P. This is called the Law of Excluded Middle.

Take the previous examples. You may say again that there is no need to accept either statement. The horse is partly brown and partly not, etc. But the whole trouble in all the cases comes through not being perfectly explicit about the terms. Which part are you talking about? That part must either be brown or not.

Again, it may be said that it is simply meaningless either to affirm or to deny, *e.g.* that virtue is white. But it seems simplest in such a case to say that virtue is not white. That does not imply that virtue is coloured.

Is there any reason for this law in addition to the previous one? The law of non-contradiction says that both statements cannot be true. But if our knowledge extended no further than this, there would still remain the possibility that neither was true. The law of excluded middle is therefore necessary, in order to negative this possibility. The two laws can be thought of together: Of the two propositions "S is P," "S is not P," one must be true and one false.

5. Suppose we are talking of a definite individual thing: a rose, a pen, a book. Let us take the following statements:

This rose is red.	This rose is not red,
This pen is broken.	This pen is not broken.
This book is on the table.	This book is not on the table.

Here we have pairs of statements, one affirming, one denying, the same thing of each subject. We have seen that one of each pair must be true and one false.

6. Suppose, for the sake of simplicity, that we have five roses together: we will call them a, b, c, d, e. We have the following pairs:

a is red.	a is not red.
b is red.	b is not red.
c is red.	c is not red.
......	....
......	....

If I say, "All the roses are red," I mean simply to assert all the first set. Now, if I say, "Some of the roses are red," do I assert *any* of the first set? Do I, for instance, assert that a is red? or that b is?

It seems not. What then do I assert ? I assert that *either* a is, *or* b is, *or* c is, etc.

Suppose I am told that b is not red, and d is not red, and e is not red. Will that make my statement false ? Clearly not, for I can still fall back on the possibility that a and c are red. How then could my statement be proved false ? Only by cutting off even these possibilities : in other words, by showing that none of them are red. The statement that some of the roses are red can be contradicted only by the statement that none of the roses are red.

7. Let us look at this again. If a, c, and d are red, then it follows that " Some of the roses are red " is true. But if " a is red " is true, then " a is not red " is false. If " c is red " is true, then " c is not red " is false. So with " d is not red." It would seem then that " Some of the roses are not red " is false. But if this were so, then it would be false to say that b and e are not red ; and hence b and e would have to be red simply because a, c, and d are. It follows from this, that the treatment of true and false propositions differs.

If $$\left.\begin{array}{l} a \text{ is red,} \\ c \text{ is red,} \\ d \text{ is red,} \end{array}\right\} \text{is true,}$$

then " Some roses are red," is true ;

but if $$\left.\begin{array}{l} a \text{ is red,} \\ c \text{ is red,} \\ d \text{ is red,} \end{array}\right\} \text{is false,}$$

then we cannot say that " Some roses are red," is false.

We can only add *true* propositions in this way.

8. We can now deduce certain results as regards the particular and universal propositions.

If " All S is P " is false, what happens ?

$$\text{" All } S \text{ is } P \text{ " means,} \begin{cases} S_1 \text{ is } P. \\ S_2 \text{ is } P. \\ S_3 \text{ is } P. \\ S_4 \text{ is } P. \\ \ldots\ldots \\ \ldots\ldots \end{cases}$$

Do we mean that every one of these statements is false ?

No. We only mean that at least some of them are false. Let *e.g.*

$$\left.\begin{matrix} S_2 \text{ is } P \\ S_4 \text{ is } P \\ \ldots\ldots \\ \ldots\ldots \end{matrix}\right\} \text{ be false.}$$

Then by the law of excluded middle,

$$\left.\begin{matrix} S_2 \text{ is not } P \\ S_4 \text{ is not } P \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \end{matrix}\right\} \text{ are true.}$$

Hence, " Some S is not P " is true.

9. We can state this systematically as follows :

If " All S is P " is false, then it is false to assert *all* the propositions :

$$\begin{matrix} S_1 \text{ is } P, \\ S_2 \text{ is } P, \\ \ldots\ldots \\ \ldots\ldots \end{matrix}$$

Hence some of these propositions are not true. It follows that some of the propositions :

$$S_1 \text{ is not } P,$$
$$S_2 \text{ is not } P,$$
$$\text{etc.,}$$

are true. Hence " Some S is not P " is true.

If " No S is P " is false, then it is false to assert *all* the propositions :

$$S_1 \text{ is not } P,$$
$$S_2 \text{ is not } P,$$
$$\text{etc.}$$

Hence some of these propositions are not true. Hence, by the law of excluded middle, some of the propositions :

$$S_1 \text{ is } P,$$
$$S_2 \text{ is } P,$$
$$\text{etc.,}$$

are true. Hence " Some S is P " is true.

If " Some S is P " is false, then it is false to assert *any* of the propositions :

$$S_1 \text{ is } P,$$
$$S_2 \text{ is } P,$$
$$\text{etc.}$$

That is, all these propositions are false. Hence, by the law of excluded middle, all the propositions :

$$S_1 \text{ is not } P,$$
$$S_2 \text{ is not } P,$$
$$\text{etc.,}$$

are true. Hence " No S is P " is true.

The student should show that if " Some S is not P " is false, then " All S is P " is true.

10. The following results may be shown :

(*a*) If " All *S* is *P* " is true, then " Some *S* is *P* " is also true. " Some *S* is not *P* " is false, and hence also " No *S* is *P* " is false.

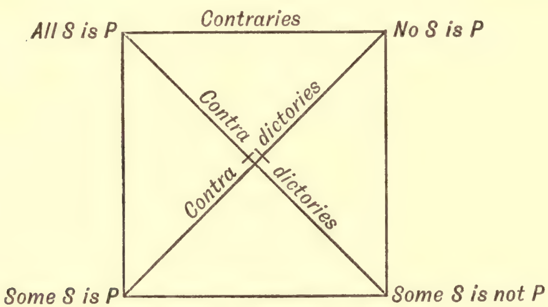
(*b*) If " No *S* is *P* " is true, then " Some *S* is not *P* " is also true. " Some *S* is *P* " is false, and hence also " All *S* is *P* " is false.

(*c*) If " Some *S* is *P* " is true, then " No *S* is *P* " is false. " All *S* is *P* " may be true. " Some *S* is not *P* " may be true.

(*d*) So for " Some *S* is not *P*."

Any two propositions which cannot both be true are said to be *incompatible*. If in addition, neither need be true, they are said to be *contrary*. *E.g.* All *S* is *P*, No *S* is *P*. But if one of them must be true, they are said to be *contradictory*. *E.g.* All *S* is *P*, Some *S* is not *P*. No *S* is *P*, Some *S* is *P*.

These relations may be represented on a diagram :



Contrary propositions are incompatible propositions, neither of which need be true. Contradictory propositions are incompatible propositions, one of which must be true.

EXERCISES.

1. If "Some *A* is *B*" is true, prove by means of the laws of thought, that "No *A* is *B*" is false.

2. If "All *A* is *B*" is true, show that "No *A* is *B*" is false.

3. If it is absurd to suppose that "All birds communicate by means of language," what is true? If it would be no less absurd to deny the falsity of the assumption that "No birds can count," what is true?

4. Define contraries and contradictories. What is the contradictory of "The book is red?" Give a statement which is contrary to this, and show why.

5. Do the statements "This horse is partly brown," "This horse is not partly brown," contradict one another? (Put both into strict logical form.)

6. What is meant by "a contradiction in terms?"

CHAPTER IX.

THE SYLLOGISM.

1. In order to explain completely the significance of the universal proposition, we must have recourse to a further principle, the principle of the *syllogism*, which can be expressed thus :

If two propositions are so related that the second is true provided the first is, and if further the first is true, then the second is true.

E.g. If A is B then A is C (a).
But A is B (b).
$\therefore$ A is C (c).

Here (a) states that the two propositions "A is B," "A is C" are so related that "A is C" is true provided "A is B" is true ; (b) states that "A is B" is true ; hence by the principle of the syllogism we are enabled to state (c) that "A is C" is true ; *i.e.* to assert "A is C." The three taken together are called a *syllogism*. (a) which asserts the connection between the two propositions, is called the *major premiss* ; (b) which asserts that the first proposition is true, is called the *minor premiss* ; (c) is the conclusion which follows from, or can be inferred

from, the two premisses. It is called simply the *conclusion*. In passing from premisses to conclusion we are said to make a *deductive inference*.

2. The major premiss may take various forms: it may be either a categorical, hypothetical, or a disjunctive proposition: corresponding, we get the categorical, hypothetical, or disjunctive syllogism.

This statement requires some explanation; for it would seem as if the only possible syllogism ought to be the hypothetical. Its general statement would be:

If A is B, then C is D.
But A is B.
$\therefore$ C is D.

But take, *e.g.*, the following premisses:

All A is B.
But X is A.

"All A is B" conveys among other things, the information: "If anything is A then it is B"; and thus in this case, If X is A, then it is B.

We have then:

If X is A, then it is B.
But X is A.
$\therefore$ X is B.

We can thus regard "All A is B" as the major, and "X is A" as the minor premiss, of a syllogism whose conclusion is "X is B."

3. How much information does "All A is B" convey? It tells us at least:

(*a*) If anything is A, then it is B.
(*b*) If anything is not B, then it is not A.
(*c*) But a thing can be B without being A.

Hence, (1) If we are told that a thing is B we cannot say whether it is A or not. (2) If we are told that a thing is not A we cannot say whether it is B or not.

Thus we have four cases :

All A is B.	All A is B.
X is A.	X is not A.
$\therefore$ X is B.	No conclusion.
All A is B.	All A is B.
X is B.	X is not B.
No conclusion.	$\therefore$ X is not A.

4. The reader can discover for himself the interpretation of the proposition "No A is B" : leading to the four cases :

No A is B.	No A is B.
X is A.	X is not A.
$\therefore$ X is not B.	No conclusion.
No A is B.	No A is B.
X is B.	X is not B.
$\therefore$ X is not A.	No conclusion.

5. Thus if the major is a universal categorical proposition, there are eight possible sets of premisses, of which four give us a conclusion. The student should get into the habit of thinking out each on its own merits, somewhat as follows :

All A is B.
X is not B.

The minor tells us that something is not B. The major tells us that if it is not B, it is not A. Hence X is not A.

Exercises.

1. All voters have a share in the management of the country. What can we conclude from the knowledge (*a*) that a person is not a voter, (*b*) that a person has no share in the management of the country, (*c*) that a person has a share in the management of the country?

2. If it is false that a good education is sometimes thrown away on a man, what conclusion can you draw from the statement that a particular man had a good education?

3. "There is a cab passing." "How do you know?" "Because of its noise. All cabs make that noise." Would this reply be correct?

4. Discuss the following, putting them into strict form:

(*a*) Wherever you have P you have Q. But in R, Q is absent. Hence in R, P is absent.

(*b*) This district has not a great rainfall, for mountains are needed before you have rain, and here there are no mountains. In (*b*) what conclusion could be drawn from the information that there were mountains in the district?

(*c*) This clause is a subordinate clause, for it begins with a relative pronoun, and any clause which begins with a relative pronoun is subordinate.

6. The importance of this discussion arises from the fact that every pair of categorical propositions which have a common term is a potential pair of premisses for a syllogism. It may be possible to draw a conclusion. Let us see to how many cases our forms will apply.

Take the syllogism

$$\begin{array}{c} \text{All } A \text{ is } B. \\ X \text{ is } A. \\ \therefore X \text{ is } B. \end{array}$$

Suppose "X is A" is true for every member of the class C; *i.e.* suppose C_1 is A, C_2 is A, etc. (*i.e.* All C is A). Then we have the set of syllogisms :

$$\begin{array}{c} \text{All } A \text{ is } B. \\ C_1, C_2, C_3, \ldots \text{ is } A. \\ \therefore C_1, C_2, C_3, \ldots \text{ is } B; \end{array}$$

or in other words, All A is B, All C is A ; $\therefore$ All C is B. If, however, we only know that the statement "X is A" is true for some members of the class C, then we have :

$$\begin{array}{c} \text{All } A \text{ is } B. \\ C_1 \text{ or } C_2 \ldots \text{ is } B. \\ \therefore C_1 \text{ or } C_2 \ldots \text{ is } B \text{ (\textit{i.e.} Some } C \text{ is } B). \end{array}$$

or in other words, All A is B, Some C is A ; $\therefore$ Some C is B. The same thing holds of all the other syllogisms.

We now have, corresponding to each of our original four sets of premisses from which a conclusion could be drawn, two other sets, making twelve different sets in all. But the argument in the new cases is precisely the same as in the original ones, *e.g.* :

$$\begin{array}{c} \text{All } A \text{ is } B. \\ \text{No } C \text{ is } B. \end{array}$$

The minor tells us that certain things are not B :

the major tells us that if things are not *B* then they are not *A*. Hence

No *C* is *B*.

Take a concrete case :

No badly-managed railways are profitable.

Some railways in the North of England are profitable.

The minor tell us that certain things are profitable. The major tells us that if they are profitable then they are not badly-managed railways. Hence,

Some railways in the North of England are not badly-managed.

It is clear that in order to treat these syllogisms with ease and accuracy, the important thing to know is the meaning of the two propositions "All *A* is *B*," "No *A* is *B*."

Exercises.

1. No honest man runs away on the appearance of a policeman. But some of these men ran away.

What conclusion can you draw ? What could be said of those who did not run away ?

2. What conclusion can be drawn from the following ?

All the history books go on one shelf. But some of these are not history books.

All the children who attend a full year receive a certificate. None of these children receive a certificate. (Do this twice, choosing (*a*) the affirmative, (*b*) the negative proposition, as major.)

In all equilateral triangles the bisector of any angle bisects the opposite side. But all triangles are not equilateral.

All good lessons are directed to one end—the development of initiative. But a lesson in which the children are told everything does not develop their initiative.

It is not always the best man who wins. But the man who wins is generally regarded as the best man.

7. All the syllogisms with which we have so far dealt have certain common characteristics (which can be discovered by an examination of the simplest four types), namely,

(*a*) The major premiss is a universal proposition.

(*b*) The common term is the *predicate* of the minor premiss.

(*c*) The subject of the minor premiss is also the subject of the conclusion.

Further we have dealt with all the possible syllogisms which present these characteristics.

It follows that if we can put any syllogism into this form, we shall be able to treat it. The problem of dealing with all future syllogisms then reduces to that of throwing them into this form. And that, fortunately, is a very simple matter.

(*a*) If we have two propositions with a common term, both of which are particular, then of course we cannot deal with them by our present method. We can deal only with such pairs as have one proposition universal.

(*b*) If only one proposition is universal, it must be chosen as major. If both are universal, either may be chosen as major.

(*c*) The next thing is, that the common term must be the predicate of the minor. But suppose it is not. Then we must find a proposition which we can substitute for the minor premiss, and whose predicate is the middle (*i.e.* common) term. Let us take an example :

All A is B.
All A is C.

Here the first proposition, being universal, can stand as major premiss. But the common term A is the subject of the minor premiss. We want it as the predicate. What we do is to replace " All A is C " by a proposition whose truth follows from the truth of that, and which has C in the subject and A in the predicate.

8. Let us take our old propositions " All S is P," " No S is P," " Some S is P," " Some S is not P," and see if we can deal with them all in this way. We leave it to the reader to prove,

(1) If All S is P, then Some P is S.
(2) If No S is P, then No P is S.
(3) If some S is P, then Some P is S.

This process of passing from a given proposition to a proposition whose truth follows from that of the given proposition, and whose subject is the predicate and predicate the subject of the given proposition, is called *conversion*.

The case of " Some S is not P " presents difficulties. Can we infer that " Some P is not S " ? No, for it is possible that " All P is S " (*E.g.* " Some animals are not men."). Can we infer that " Some P is S " ? No, for it is possible that " No P is S."

(*E.g.* wherever "No S is P.") Hence it seems that in this case we are powerless.

We get out of the difficulty by a device. Whatever is not P, is characterised by the absence of the properties P. If "Some S is not P," then "Some S is characterised by the absence of P." It follows that "Something characterised by the absence of P is S." We have now succeeded in getting S in the predicate; though only by passing from P to a new term.

If we symbolise the class characterised by the absence of P (all things which are not P) as the class not-P, we can write the process thus:

Some S is not P. $\therefore$ Some S is not-P.

This can now be converted: Some not-P is S.

The device of passing from one proposition to another whose truth follows from that of the first, and whose predicate is the contradictory of the predicate of the first, is called *obversion*. It may be practised on all the propositions.

Examples.

1. Convert the following propositions:
   Some men are good runners.
   None of the lower animals can communicate by means of language.
   Twenty men killed twenty tigers.
   All trees have leaves.
   A man sometimes faces misfortune bravely.

2. Obvert the following:
   No men who are traitors are to be spared.
   All men who can handle a rifle are to go.
   All S is P.

3. Convert "No not-A is B." Then obvert. Hence show that "Only A is B" is equivalent to "All B is A."

4. All things are either clever or stupid or neither clever nor stupid.

Things.

Clever. Stupid. Neither clever nor stupid.

Show that

(*a*) If X is not clever we do not know whether it is stupid.

(*b*) If X is clever we know that it is not stupid.

5. Can we pass (*a*) from "All knowledge is useful" to "No knowledge is useless," (*b*) from "All X speak well" to "No X speak badly," (*c*) from "No X speak well" to "All X speak badly."

6. From the following, deduce propositions in which the new predicates are the same as the old subjects:

Some men are not good runners.

A man who has lost his employment through an accident is not always incapacitated for lighter employment.

All is not gold that glitters.

9. We are now able to carry out the two rules:

(1) A universal premiss must be chosen as major.

(2) The common term (generally called the *middle* term) must be made the predicate of the minor.

For example: All men are rational: but some men do not reason correctly. What conclusion follows?

Applying the rules, we have:

Major: All men | are | rational.

Minor: Some men | are not | persons who reason correctly.

The middle term is men. We must replace the minor by a proposition in which men is the predicate. We must first obvert.

Some men | are | persons who do not reason correctly.

We can now convert this.

Some persons who do not reason correctly | are | men.

This is our new minor.

Major: All men | are | rational.

Minor: Some persons who do not reason correctly | are | men.

The minor tells us that some things are men. The major tells us that if a thing is a man then it is rational. Hence,

Some persons who do not reason correctly | are | rational.

What could we say about rationals? Convert. We get,

Some rationals | are | persons who do not reason correctly.

Or, obverting,

Some rationals | are not | persons who reason correctly.

We have now reached a proposition containing the original terms.

10. One rule remains. If both premisses are universal, either may be chosen as major; but if when one is chosen as major we can derive no conclusion, we must not infer that no conclusion is possible until we have tried the other as major.

There are only three cases where this happens.

(*a*) All A is B. (*b*) All A is B. (*c*) All A is B.
All B is C. No A is C. No C is A.

(*a*) All A is B.
All B is C.

Choose "All A is B" as major. The middle term is B. This must be the predicate of the minor. Now if All B is C, then Some C is B.

We have now,

All A is B.
Some C is B.

The minor tells us that something is B; but the major tells us that if a thing is B, we do not know whether it is A. Hence we do not know whether some C is A.

Hence no conclusion, when "All A is B" is taken as major.

Try "All B is C" as major. Then "All A is B" is minor, and B is already the predicate. Thus we have

All B is C.
All A is B.

The minor tells us that something is B: the major tells us that if a thing is B, then it is C. Hence All A is C.

(*b*) All A is B.
No A is C.

Taking All A is B as major, we have,

All A is B. All A is B.
No A is C. $\rightarrow$ No C is A.

The minor tells us that something is not A. But from the major, if a thing is not A, we do not know

whether it is B. Hence we do not know whether C is B. Hence we get no conclusion, when "All A is B" is chosen as the major.

Taking "No A is C" as major, we have

$$\begin{array}{ll} \text{No } A \text{ is } C. & \text{No } A \text{ is } C. \\ \text{All } A \text{ is } B. \rightarrow & \text{Some } B \text{ is } A. \end{array}$$

The minor tells us that something is A : from the major, if a thing is A, then it is not C. Hence Some B is not C.

Similarly with (*c*).

Exercises.

1. What conclusions can be drawn from the following :

   It is as absurd to suppose that all men are capable of exercising an intelligent use of the vote as it would be to deny that all men use the vote.

   His books are of two kinds : those he wrote to please himself, and those he wrote to earn a living. Now work written for the joy of writing is good work ; while work written merely to earn a living is bound to be of inferior quality.

   All good lessons develop initiative. But lessons in which children are told everything are not good lessons.

   Only men are rational ; only rational beings can use language. (It is generally most convenient, in syllogistic work, to put Only S is P into its equivalent form All P is S).

   All men are rational : only men can use language.

2. In many arguments in ordinary life, one premiss is omitted. For instance, in An indefinable something about him proclaimed him to be a gentleman, the two premisses are, He had an indefinable something about him ; All persons with this indefinable something about them are gentlemen. Point out the implied premisses in the following :

Your saying that proves your want of tact.

It was only from his speech that you could tell that he had begun life in humble circumstances.

A historian cannot judge accurately of contemporary events, for their consequences have not yet appeared.

Goethe was a poet. Goethe was a scientist. Hence poetry and science are not incompatible. (Here we seem to have two premisses and a conclusion; but the two propositions must be combined into one premiss and an implied premiss supplied. Let us look at it roughly. Thus: Goethe was both a poet and a scientist, *i.e.* poetry and science were combined in Goethe. The implied premiss is now clear. No qualities which are combined in a single individual are incompatible.)

It is false to suppose that some children are utterly stupid: but most children show a lack of intelligent interest in Latin; hence interest in Latin is not a test of intelligence. (Combine the first two propositions in a syllogism.)

3. In some cases the premisses have to be altered in form before we can draw a conclusion. *E.g.*

No English railways are badly managed; No badly managed railways are profitable:

we can draw no conclusion as the premisses stand; but we can legitimately infer from the second premiss that All badly managed railways are unprofitable. We can now draw a conclusion.

Treat the following:

No men are immortal; No men are omniscient.

No immortals are men; no omniscient beings are men.

A revolutionary government is never successful for long; no Conservative government is revolutionary.

4. Put the following arguments into proper form:

Whatsoever their estates be, their house must be fair. Therefore from Amsterdam they have banished sea-coal, lest it soil their buildings.

Most of the rivers flow across the Meseta to the west. As a rule they run into rocky channels cut deep into the plateau, and are, therefore, of little use for navigation.

On this west shore we found a dead fish floating, which had in his nose a horn, straight and torquet, of length two yards lacking two inches, being broken in the top, where we might perceive it hollow, into which some of our sailors putting spiders they presently died. I saw not the trial thereof, but it was reported unto me of a truth, by the virtue whereof we supposed it to be the sea-unicorn. (Hakluyt.)

11. We have now discussed all the cases except those in which neither premiss is universal; *i.e.* in which both premisses are particular. In these cases the principle of the syllogism does not apply. From a particular proposition we cannot derive a proposition which expresses the required connection between two propositions. Hence we cannot use the syllogism. Whether a conclusion can be derived in any other way we do not here ask.

12. Hypothetical syllogism. Here the major is a hypothetical proposition. The general form of the major is, "If *A* is *B*, then *C* is *D*." "If a country is mountainous, it will have a large rainfall." "The match will light if it be struck." (This should be written in the form, "If the match be

struck, then it will light".) In a hypothetical proposition, the proposition expressing the condition is called the antecedent ; that asserted under this condition, the consequent.

Let us look at one of the above propositions. "If a country is mountainous, it will have a large rainfall." Suppose we assert that a country is mountainous. Then we must go on to assert that it has a large rainfall. Suppose the country is not mountainous. Then clearly we do not know whether it has a large rainfall. If we are told that a country has a large rainfall, can we say whether it is mountainous or not ? Does our hypothetical proposition assert the *only* conditions under which a country has a large rainfall ? No. There may be other conditions. Hence we cannot say, in this case, that the country is mountainous. But if, finally, the country has not a large rainfall, then it cannot be mountainous : for if it were mountainous, it would have a large rainfall. This analysis is exactly parallel to that of the meaning of "All A is B" in Section 3.

So in the case of "If A is B, then C is D." If we know that A is B, then we know further that C is D. If we know that A is not B, then we do not know whether C is D. If we know that C is D, then we do not know whether A is B. Finally, if we know that C is not D, we know that A is not B ; for if A were B, then C would be D.

Thus the two possible syllogisms are :

If A is B, then C is D.	If A is B, then C is D.
But A is B.	But C is not D.
$\therefore$ C is D.	$\therefore$ A is not B.

From the affirmation of the antecedent you can affirm the consequent, and from the denial of the consequent you can deny the antecedent.

Suppose we are given the proposition :

If you do your work, you will pass the examination.

What follows (*a*) where you pass, (*b*) where you do not pass, (*c*) where you do not do your work ?

13. Disjunctive syllogism. Here the major is a disjunctive proposition. The general form of the disjunctive proposition is, "A is either B or C." Here the question arises as to the meaning of "either....or." We certainly mean that A must be one of the two, but do we mean that it cannot be both ? Sometimes we do, and sometimes we do not. *E.g.* in "He is either thirty or very nearly so," clearly the alternatives exclude one another. In "Applicants must be graduates in either arts or science," we do not mean to exclude a graduate in both.

Thus the proposition may mean,

(1) A is either B or C (perhaps both).

(2) A is either B or C (not both).

The second tells us more than the first ; in the absence, therefore, of any definite indication as to which is intended we must take the first meaning. The simplest plan is to state all the possible alternatives in such a way that they exclude one another; there is then no doubt as to the interpretation of "either....or."

Thus (1) would be : A is either (B and not C) or (C and not B) or (B and C).

(2) would be : A is either (B and not C) or (C and not B).

We have then the following syllogisms :

A is either B or C.	A is either B or C.
But A is not B.	But A is not C.
$\therefore A$ is C.	$\therefore A$ is B.

If the alternatives exclude one another, we have also :

A is either B or C.	A is either B or C.
But A is B.	But A is C.
$\therefore A$ is not C.	$\therefore A$ is not B.

14. A syllogism whose major is a hypothetical and whose minor is a disjunctive proposition is called a dilemma. In ordinary speech we are said to be in a dilemma when presented with two alternatives, each of which leads to unpleasant consequences— the *horns* of the dilemma. *E.g.* We must do either P or Q. If we do P, X follows ; if Q, then Y : hence either X or Y must follow. When X and Y are equally unpleasant, we are said to be in a dilemma. A little girl, on being given two apples, one much bigger than the other, for herself and her brother, extricated herself from the dilemma (and presented it to her brother) by offering him the choice, asking "Is you greedy ?" His dilemma was (expressed formally), "If I take the larger one I shall be thought greedy, and if I take the smaller I shall have less than my share. But I must take either the larger or the smaller ; hence I shall either be thought greedy or have less than my share." In Logic the word has a wider meaning, being applied to various complex syllogisms whose major is a complex hypothetical and whose minor is a disjunctive proposition : although, in general, the

various alternative consequences are similar in nature.

15. To point out things—to state a case—is not reasoning in the strictly formal sense. It is a massing of facts and general statements in such a way as to produce in the mind of the hearer a willingness to look at a particular question from a certain point of view. It is like saying, "This is the way the thing presents itself to me—these are the considerations which weigh most with me in discussing the question." This is not formal reasoning: but it is a preparation for formal reasoning. You are left to draw your own conclusion; or are put in such a position that when the speaker draws his conclusion, you will be ready to listen to him, and to accept his conclusion as the correct one.

This is not formal reasoning: but its effect on us has the force of formal reasoning. Hence if you were asked to pick out a passage which showed argumentation, you would probably light on one such as this.

The only formal reasoning proper is the drawing of conclusions: where we have explicit or implicit, a "therefore," "hence," "consequently," "it follows that," "we conclude that," etc.; or a "because," "since," "for," etc.

Take a few passages of controversial literature and see how great a proportion the preparation bears to the drawing of conclusions. The present passage exemplifies the same thing.

If this is so, it must be because it is the only effective mode of argument. You cannot draw conclusions from a complicated set of circumstances

until you look at them from a certain point of view; and your conclusion will vary according to the point of view you adopt. Every question is complicated: it presents various aspects. You have only to read the letters in the daily paper, or the parliamentary debates, or the controversial literature of any question which interests you, to see how each person has his own way of viewing the facts, and draws his own conclusion therefrom. This should not make us despair of ever arriving at the truth: but it should make us extremely careful to endeavour always to look at a question from all sides. And it shows us clearly the limits of Formal Logic. For this weighing of considerations is the most important and the most difficult part of actual thinking, and the value of the conclusions drawn depends entirely on the way in which this has been done. Now Formal Logic does not deal with this part of the process.

EXERCISES.

1. Put into logical form the following:

   It is as absurd to affirm that birds can communicate by means of language as it is to deny that they can count; whence it may be inferred that counting does not depend on the use of language. (By our ordinary methods we reach the proposition: Some animals which can count are not animals which can communicate by means of language. An implied premiss is then necessary. It is simplest in this case to put the implied premiss into hypothetical form.)

2. In the following give a rough account of the steps of the argument, and finally put the argument into strict logical form.

(*a*) When the worldly-wise Chesterfield gives the advice "Never tell stories," he has in view the social bore. He is pleading for the rights of the individual in conversation, which are always endangered when story-telling creeps in. The teller of tales is of necessity a monopolist.

In expository work, whether in school or on the platform, the speaker's monopoly is already granted, so any objection to story-telling must be based on other grounds. (Adams, p. 247.)

(*b*) Birmingham is not on the coal-field, so that the actual smelting of iron is not carried on there. It specialises in the manufacture of all kinds of metal goods, including machinery, pins and needles, screws, guns, and ammunition. Birmingham is a long way from any important seaport, so that it cannot compete with other places in the cruder processes of manufacture, where large quantities are handled at cheap rates. The value of the workmanship of its products counts for more than the value of the raw material, and therefore slightly increased transport rates are insignificant compared with the total value of the articles. (Mort, p. 35.)

(*c*) There was an usage in England, and yet is in divers countries, that the nobleman hath great franchise over the commons and keepeth them in servage, that is to say, their tenants ought by custom to labour the lords' land, to gather and bring home their corns, and some to thresh and to fan, and by servage to make their hay and to hew their wood and to bring it home. Thus the noblemen and the prelates

are served by them, and specially in the county of Kent, Essex, Sussex, and Bedford. These unhappy people of these said countries began to stir, because they said they were kept in great servage, and in the beginning of the world, they said, there were no bondsmen, wherefore they maintained that none ought to be bond, without he did treason to his lord, as Lucifer did to his God; but they said they could have no such battle, for they were neither angels nor spirits, but men formed to the similitude of their lords, saying why should they then be kept so under like beasts; the which they said they would no longer suffer, for they would be all one, and if they laboured or did anything for their lords, they would have wages therefor as well as other. (Lord Berners, *Insurrection of Wat Tyler.*)

(*d*) Clay will float in water and only slowly settles down. Is this because clay is lighter than water? Probably not, because a lump of clay seems very heavy. Further, if we put a small ball of clay into water it at once sinks to the bottom. Only when we rub the clay between our fingers or work it with a stick—in other words, when we break the ball into very tiny pieces—can we get it to float again. We therefore conclude that the clay floats for so long not because it is lighter than water, but because the pieces are so small. (Russell, p. 9.)

(*e*) It is an undertaking of some degree of delicacy to examine into the cause of public disorders. If a man happens not to succeed in such an enquiry, he will be thought weak and visionary; if he touches the true grievance, there is a danger that he may come near to persons of weight and consequence, who will rather be exasperated at the discovery of their

errors, than thankful for the occasion of correcting them. If he should be obliged to blame the favourites of the people, he will be considered as the tool of power; if he censures those in power, he will be looked on as an instrument of faction. But in all exertions of duty something is to be hazarded. (Burke.)

3. Build up the following facts into a single connected composition, arranging them in what you consider the best order:

The swallow has very long wings—lives on insects—has great powers of flight— feeds in the air—its feet not adapted for walking or climbing—has a broad soft bill—migrates to warmer lands in winter—does not eat fruits or grubs—there are no insects in the air of Britain in winter—the swallow is not swifter in flight than the falcon or pigeon-carrier, but can turn and wheel more rapidly—has a long forked tail. (Lower Leaving Certificate Examination, English, 1906.)

4. Build up the following statements into a connected composition. Arrange the sentences in what you think the best order, avoiding needless repetitions.

Britain is one of the greatest powers of the world—great in industry and commerce—her industrial supremacy threatened by Germany and the United States—English language widespread—colonies a source of strength—rise of new powers (*e.g.* Japan)—colonies numerous—risk of population crowding into towns—colonies favourably situated—certain elements of danger—competition for trade growing keener—yeoman class dwindling—Britain no longer the workshop of the world—her people highly civilised—other great empires have risen and fallen—the prosperity

of a country depends chiefly on the energy and patriotism of its citizens and their power of adapting themselves to new conditions—the future of the British Empire is an interesting speculation. (1907.)

Discuss your results, from the point of view of logical order.

CHAPTER X.

FURTHER CONSIDERATIONS RELATING TO THE JUDGMENT.

1. The material with which Logic deals is, as we have shown, the judgment. And one of the most fundamental properties of a judgment is, we have seen, that it must be either true or false. This is connected with its most important characteristics, namely :

(*a*) It is based on grounds.

(*b*) It has consequences.

A judgment based on true grounds is true ; any consequences a true judgment has are true. But it is possible to put forward false grounds for a true judgment : *i.e.* the fact that a judgment is defended on false grounds does not prevent the judgment itself from being true. But if we give false reasons for a judgment which we know otherwise is true, it seems clear that we do not know the whole truth about the matter : *e.g.* I may know by observation that a particular event has happened, but my account of the reasons for its happening may be quite incorrect. Here I may be said to know the fact, but not to judge with entire correctness con-

cerning it. For complete knowledge, not only must our beliefs be true in themselves, but they must be correctly based : in other words, the reasons we give for them must be correct.

2. If two or more true propositions can be joined together in such a way that a conclusion can be derived, then the conclusion is true. It follows that all the propositions which can be deduced from a set of true propositions are themselves true. They will thus harmonise : in other words, no two will conflict.

But if from a set of propositions claiming to be true, two propositions can be deduced which contradict each other, then some of the original set must be false.

This furnishes us with a preliminary method of distinguishing between truth and falsehood ; *e.g.* if the account given by a witness in a court of law is plausible (*i.e.* if it does not conflict with what could be expected to happen) and if the statements do not contradict one another at any point, then it can be accepted as at least likely to be true. On the other hand, if on further cross-examination the witness makes statements which contradict what he has previously said, then he is clearly not telling the truth.

3. Thus, judgments cannot stand alone. They must justify themselves when brought into contact with all true judgments.

Let us look at this for a moment, in connection with the way in which statements are made in ordinary life. We shall find that very often statements are made by us apparently in all generality, whose

consequences we are not willing to accept. This is because in ordinary life we are not concerned to be perfectly accurate. We make statements which are not wholly true, with certain unexpressed reservations as to the nature of the cases to which they will apply. We probably could not express clearly the limits within which we are willing to let our statement apply, but we have a feeling as to what these limits are. In fact, we may only make the statement with the object of drawing attention to certain features of the case before us, which we take to be the important ones. Our analysis, being only for practical purposes, is neither exact nor exhaustive. This is especially the case in our use of proverbial expressions. *E.g.* watching a person who has narrowly escaped being run over, we may come to the conclusion that his danger was due mainly to his hesitation as to whether to "stand or go": and we may express this feeling by saying "The man who hesitates is lost."

Various proverbs may conflict. "A penny saved is a penny gained" has to be corrected by "Penny wise, pound foolish." But the same person would accept both, because he accepts neither without certain reservations.

The laws of a country are general statements which have to be "interpreted": *i.e.* which are not intended to be accepted without restriction. We speak about obeying the spirit rather than the letter of the law, or breaking a law in spirit while keeping to it in letter. These phrases are significant only because every law is subject to certain reservations, and is not absolutely or unreservedly binding.

4. All this shows (*a*) with how great care the general propositions in practical life have to be interpreted, (*b*) how much labour in sifting out unexpressed qualifications has to be undertaken by anyone who desires to formulate truths which will be completely true. In the affairs of practical life, completely true propositions are almost an impossibility : in science they involve years of co-ordinated labour. It is easy to make yourself generally intelligible without being precise : it is extremely difficult to attain precision.

5. Certain so-called fallacies are due to neglecting the implied qualifications, and deducing consequences. *E.g.*

All successful business men are lucky.

No man who breaks his leg is lucky.

∴ No man who breaks his leg is a successful business man.

i.e. No successful business man breaks his leg.

Here, it is clear, we mean the word lucky to be taken in a limited sense (and a different one) in each case.

6. If a judgment claims to be completely true, both it and all its consequences must be accepted, however it may be confronted with other true propositions. A proposition which will not stand this test is not completely true. A true proposition then can never be altered : true once, true always, at all times, and under all circumstances. This condition which must be satisfied by all completely true propositions is called the *Law of Identity*.

7. Every judgment, we have seen, is based on grounds. Its truth depends on their truth. Now we showed (p. 35) that we do in ordinary thinking

prove a particular result by showing the application of a general rule, which we accept as true. We left for further consideration the question as to the justification of these general rules. We get them, it was suggested, by a gradual process, of comparison and classification of instances, and the formation and testing of hypotheses. We have now to discuss this process more closely. We have *two* questions to ask. (1) What is the process by which we arrive at our generalisations? (2) What kind of *proof* have we for them?

EXERCISES.

1. Peacock speaks of "premisses assumed without evidence or in spite of it, and conclusions drawn from them so logically that they must necessarily be erroneous." Is it possible that the conclusions should be true? If so, would the reasoning be valuable?

2. Is it possible for all the following to be correct? All R is S. All M is Q. All Q is R. No M is S.

3. What conclusion can you draw from the following:

(*a*) All cats are dogs.
All dogs are mortal.
∴ All cats are mortal.
(*b*) All rabbits are cats.
All cats dwell in holes.
∴ All rabbits dwell in holes.

(Note whether the propositions are true or false.)

4. Put into logical form the following:

The sheep has wool because it was created for man's use, man needs wool for clothing, and no other animal but the sheep supplies it.

Stones fall to the ground because they are made of earth; and everything made of earth tends to join the earth.

This child is stupid because he does not attend: for all children who do not attend are stupid.

If in these the conclusion and one premiss are true, does it follow that the other premiss is true?

5. Test the following:

The taking of life is prohibited by the commandments. One who encourages the taking of life is as bad as one who actually takes life. To eat meat is to encourage the taking of life. Hence the eating of meat is prohibited by the commandments.

To be of any value, the lesson should arouse the child's interest. But a child's interests are not the same as a grown-up's. Thus what interests a child does not interest the teacher. Hence to be of any value, the lesson should not interest the teacher.

CHAPTER XI.

METHODS OF AGREEMENT AND DIFFERENCE.

1. One of the most common methods of leading children to the definition of a term (and a method which arises naturally out of our discussion) is to put before them examples of the things denoted by the term, and allow them to discover the common characteristics. In this there are various dangers.

The examples are chosen by the teacher (or, if collected by the class, at least grouped by him) with the object of throwing into prominence certain features which all have in common. But the children themselves, even though they have not been told to look for the common features, do so naturally: and their conclusions may differ entirely from those desired by the teacher. They may light on striking features which are not really essential. If, *e.g.* in defining the subject of a sentence, propositions are chosen in which the subject stands first, this characteristic may be seized on by the class as the most important thing about a subject. The teacher may, and often does, unconsciously aid in this process, since an example of one kind naturally

tends to call up examples after the same pattern. An additional danger arises from the peculiar relation between teacher and class: thus the class may choose as common characteristics, not such as strike them, but such as they think the teacher wants them to choose.

2. It follows that in giving examples of a definition, particular care should be taken to vary the illustrations. On what principle is this to be done? After we have decided what are the common characteristics which we intend to include in the definition, we should carefully examine what other characteristics are common to the most usual instances of the thing we are defining. The object of our examples must be to eliminate these. Thus the subject of a sentence is very often the first word in the sentence; it usually comes before the verb; but we do not want these characteristics to be attended to. We must then provide illustrations in which the subject is in a different position, in order to concentrate attention on the essential features of subjects.

3. The principles on which the children act unconsciously are:

(*a*) If certain features stand out as common to all the examples of an object, then they are the features by which to recognise that object.

(*b*) If there are certain features which are present in some, and absent in other, examples of the object, then they cannot be made use of in recognising the object.

These principles are used also, in slightly different form, in the attempt to discover the causes of certain

events. *E.g.* I have a headache ; and I remember that I had a headache on various occasions before, after having tried to read by the aid of the light I am at present using. In searching for the cause of my headache I am here seeking for some feature of my present surroundings, which was prominent on other occasions when I had a headache : and I decide that this is probably the cause. It is clear that this conclusion is liable to error. Many popular superstitions are kept alive through an error of this sort. We notice that on various occasions when salt was spilled, or knives were crossed, the appropriate misfortune happened. This method of reasoning is called the Method of Agreement. It may be stated symbolically as follows :

If on the various occasions on which an event E has happened, certain common features C can be discovered to have preceded or accompanied E, then these common features C may be the cause or part-cause of the event E.

Suppose children are listless in school. It is unlikely that we shall discover the cause by merely endeavouring to recall all previous cases. But if we have noticed that the children are often listless at this hour of the day, then we have a clue at once. For we can now provisionally neglect all cases except those which happened at this time, seeking for features which may be expected to be present at this hour more than at other hours, and which would help to produce listlessness.

In cases where experiment is possible, we produce certain conditions, and observe the results. If we could be sure that these conditions were all

essential, then we could argue as to cause and effect. In order to see what conditions are most likely to be essential, it is necessary to perform the experiment many times, with different materials. The simplest plan is to run several experiments together.

4. The Method of Agreement helps us mainly in cases where we have already hit upon some clue. But even in these cases it is an unsafe guide. Of course we do not attend to all the common features in the various cases ; we neglect a very large number as irrelevant. *E.g.* in the above instance we disregard the fact (common to all the instances) of pictures hanging on the walls, of the floor being made of wood of the presence of the teacher, and of the work going on in other classes. We thus neglect most of the features common to the various instances. Which ones then are we to retain ? In the end, we generally give heed only to those which seem likely to have been the causes. The history of science is full of the mistakes due to thinking that the most striking characteristic common to the various instances has been one of the main causes of an event. Heat a poker at one end, and hold it vertical, with the hot end downwards ; your hand soon becomes hot. Not so if the hot end be held upwards. This shows, said Bacon, that heat rises. In reality, the difference is due to the surrounding air. Indeed the air has often been neglected in cases where it has played the most important part : in the burning of substances, in the falling of bodies at different rates, in the growth of plants. Thus Van Helmont planted a shoot of willow, weighing 5 pounds, in a pot of soil weighing 200 pounds. After five years

the tree weighed just over 169 pounds. During that time, says Helmont, it had received nothing but water. The soil at the end of the time weighed almost 200 pounds. "Therefore," concludes Helmont, "the 164 pounds of wood, bark, and root arose from the water alone." (Quoted by Russell, *The Soil*, p. 47, who adds, "It is now known that the last sentence should read, 'Therefore the 164 pounds of wood, bark, and root arose chiefly from the water *and air*, but a small part came from the soil also.'")

Thus causes are very often very obscure, and the results reached by the Method of Agreement very liable to error.

5. In consequence we must use some method to enable us to discriminate between those common features which are, and those which are not, concerned in the production of the event. One important method is this : if when we take away any of the features which we regard as a possible part-cause the effect is no longer produced, then we can safely regard it, or something connected with it, as a part-cause. Otherwise we cannot. *E.g.* in the case of listless children, if we find that the room is generally badly-ventilated at the hour in question, but that the listlessness vanishes when we ventilate the room, the conclusion is clear. Bad ventilation was at least a large part of the cause. But we could not even yet say that it was the entire cause. To discover the entire cause, we should have to vary all the common characteristics. Part of the listlessness might have been due to the teacher's lack of interest in the lesson—which better ventilation

might either remove or counteract. And so on. In Helmont's experiment we could not say that the water was the whole cause of the tree's growth until we had varied all the other common elements; until, for instance, we had taken away the air.

This method is called the Method of Difference. The reader should formulate it symbolically for himself.

EXERCISES.

1. Contrast the following:

> I go into a room, and after examining all the books on a shelf, say, All these books are Logic books.
>
> I go into a room, and after taking at random a large number of books on a shelf, say, All the books I have examined are Logic books; I expect that all the books on the shelf are Logic books.

Which of these is similar to the process by which I conclude that all crows are black? That sugar is sweet? That all the students in a particular class passed a certain examination? Which do you think is most generally used by the scientist? Why?

2. One of the methods in the above question is called Induction by Complete Enumeration (or Simple Enumeration). One is called Induction by Incomplete Enumeration. Justify these names, and state each method symbolically.

3. A child notices that milk sometimes tastes sweet, sometimes not. Milk tastes sweet when sugar has been put in and not otherwise. Further, the more sugar is put in, the sweeter the milk tastes.

A scientist notices that across the Alps the rainfall is great on high peaks, greater on higher peaks, and less in the valleys.

A scientist notices that the more he heats a bar of iron, the more the iron expands: and that when the iron cools, it contracts.

What conclusions as to causes would be drawn in the above cases? Is there any common method employed in them? If so, can you describe it? Describe it in symbols.

4. If I have liked all the books of a particular writer I have so far read, am I justified in expecting that I shall like the rest of his books? Is this *merely* induction by incomplete enumeration? or do other considerations apply? If so, what?

5. Point out precisely where the method of agreement and where the method of difference is used, in the following:

Ten pots of wheat are kept only just sufficiently moist for growth, and ten are kept very moist but not too wet. All the pots have the same soil, and are given the same conditions of temperature, sunshine, etc. After a time it is noticed that in the first set the leaves are narrow and the plants small, but in full ear, ripe and yellow; while in the second set the leaves are wide and the plants big, but not yet ripe. (Russell, p. 70.)

6. The method in Question 3 is called the method of Concomitant Variations. What precisely do these words mean? Do they describe the method accurately? Show the application of the method in the experiment which leads to Boyle's Law regarding gases.

7. Is the method of concomitant variations used in the experiment in Question 5? Discuss whether this method can be regarded as a particular case of the method of difference.

8. Apply the method of agreement and the method of difference to show that evaporation is caused by a raising and condensation by a lowering of temperature.

9. Discuss the following mistakes made by children:

That the Highlands of a country are always in the North.

That a whole number cannot be a fraction; and that you cannot divide 25 by 100 because the divisor must always be less than the dividend.

That a Noun Clause (in direct speech) is a Principal Clause, because it conveys meaning by itself.

On being asked to make a pun, a child said, "The man caught a hare." This was a pun because "hare was a word with two meanings."

CHAPTER XII.

ANALOGY.

1. The fundamental method by which ordinary reasoning is carried on is the method of Analogy. The principle of this method is, that if two cases agree in very important respects, they agree in all respects generally connected with these. This is the principle on which a child groups objects into classes. It is clearly the principle on which we proceeded in investigating causes by the methods of Agreement and Difference. For the various cases we compared were very different in certain respects (no two cases are alike, if we take account of all their features). But we neglected the points in which they differed, assuming that the important points in which they agreed could be treated by themselves. It is the principle on which all our ordinary confidence is based. *E.g.* we go into a railway station with which we are familiar. It wears a perfectly familiar aspect. Of course, there are very many details in which its aspect to-day differs from its aspect yesterday—there may be a greater or less volume of traffic, and different people are moving about. But in all its important or essential features it is the same.

We proceed confidently (indeed mechanically) about our business—buying a ticket, and taking our seat in the train ; on the justifiable assumption that these things will present the same important characteristics as before. The principle of analogy is clearly involved in all this. Sometimes this principle leads us astray. Things look the same as they were yesterday, and we proceed confidently as if they were. If they are not, an accident happens.

2. Though sometimes the principle leads us astray, yet we could not do without it. Without it, we should be in a perfectly strange world, and should soon cease to act. We should be too bewildered by the variety which things present. Unless we could neglect the variety and attend only to certain characteristics, we should not be able to recognise objects.

The principle may be used with more or less exactness. "All our reasonings," says Hume, "concerning matter of fact are founded on a species of Analogy, which leads us to expect from any cause the same events we have observed to result from similar causes. Where the causes are entirely similar, the analogy is perfect, and the inference drawn from it is regarded as certain and conclusive. Nor does any man ever entertain a doubt, when he sees a piece of iron, that it will have weight and cohesion of parts, as in all other instances which have ever fallen under his observation. But where the objects have not so exact a similarity, the analogy is less perfect, and the inference is less conclusive ; though still it has some force, in proportion to the degree of similarity and resemblance.

The anatomical observations, formed upon one animal, are, by this species of reasoning, extended to all animals; and it is certain that, when the circulation of the blood, for instance, is clearly proved to have place in one creature, as a frog, or fish, it forms a strong presumption that the same principle has place in all."[1]

3. The use of this principle by children is very important, and its dangers should be noted and guarded against. Things which look alike are taken by them to be the same. This is especially the case in their attempts to discover the "rule" for the solution of a problem. For instance, in problems in proportion, two examples on "men working in a field" tend to be solved by the same method, because this characteristic is the one which is strikingly common to the examples. As a practical result, it follows that we should be very careful in transition from one kind of example to another, not merely because we want the student to work correctly, but because every time he applies a rule wrongly his confidence in the rule diminishes. And as a child's success in thinking depends very largely on his confidence in his tools, we should be extremely careful not to do anything to diminish it.

4. The principle is very important to the scientist. Indeed his main problem is to discover with just what characteristics he can apply the principle with safety. During the earliest investigations in a subject he has to depend largely on his own insight; but throughout he has to make very careful experiments in order to test the results he has arrived at.

[1] *Inquiry concerning Human Understanding.* Section IX.

For instance, at the present stage of our knowledge concerning life on other planets, we are compelled to trust to probable indications. Certain other planets have all the constituents necessary to support life, but we do not know just how life begins even when these constituents are present. Analogy here is infected with ignorance; and we can only say that it is very likely that in the planets so similar to ours in the things necessary to support life, there is life. But if we knew how life arose on our planet, we should be able to reason with more certainty.

In two different cylinders a gas is placed: the gas in one suddenly begins to alter in volume. What point of similarity in the two cases would lead us to expect that the gas in the other cylinder should alter in volume? Investigation shows that the two cases can be regarded as similar not on account of the shape of the cylinders, or the colour of their material, or even the nature of the gas; the important points are the pressure and the temperature. When we have discovered this, we can extend our result. The principle of analogy allows us to argue from this experiment in the laboratory, in which the scientist uses a gas enclosed in a U tube by mercury, to the result in the case of a gas contained in the cylinder of an engine. In the latter case clearly there is much more complication: but so far as the alterations in pressure, temperature, and volume are concerned, the analogy between the two is perfect. The changes in the U tube are a model, very much simplified, of the actual changes in the cylinder of the engine.

Take another case. Plants are growing all around us. They vary in different places. The leaves of the plants found in some places are broad and thick, while those of the plants found in other places are narrow and spiky. Why? The circumstances are extremely complicated; what are the differences in the two cases? One prominent difference which is noticed is perhaps, that in the former case the land is moist, and in the latter case, dry. Is this the reason for the difference? Let us construct a very simple model of the situation in the laboratory (at least, what would be a model if the difference were due to water). Let us plant various pots of (say) mustard or wheat, giving some much water, others little, others a moderate quantity; keeping them under the same conditions otherwise. Let us note the results. We discover a difference in the leaves and in the growth generally, quite corresponding to the differences in the original cases. (See Russell, p. 64-80.) The analogy between the model and the actual case is thus complete.

This use of "models" of every form is one of the most important consequences of the principle of analogy. It enables us to simplify our problems by getting rid of much of the concrete detail, which, though not affecting the question, may yet interfere with our consideration of the important features of the case. It should be clear that we cannot begin to construct a model of this sort until we have formed some theory as to what the important features of the case are. We then form a tentative model. If our theory is the right one, then our tentative model will be a real model.

This treatment of a concrete problem by means of an abstract model is fundamental in all thinking. Every problem in Arithmetic or Algebra is an instance of it on a very elementary level. For instance, the problems dealing with clocks (At what time between six and seven are the hands together? etc.) depend on our ability to make a model of the situation, including nothing but certain numerical relations. So the problem of the flight of a cannon ball, or the wearing away of rocks at a certain place, can be treated successfully only by our thinking in terms of an abstract model. The physicist or the geologist pictures the world as a battle-ground of forces of a particular nature, neglecting all its other aspects. This forms his model; and analogy allows him to argue from his model to the concrete facts.

Thus in order to discuss a concrete problem we must (*a*) pick out what seem to us the important elements, (*b*) construct an abstract model of the case where the important elements are clearly seen, and (*c*) after investigating the question by means of this model, think back into the concrete facts the result of our work on the model. In all this we proceed by means of the principle of analogy.[1]

A great investigator is shown, not merely by his ability to form theories as to the important elements in a particular case, but even more by his genius in

[1] From this follows a result which will prove of practical importance. Children are accustomed to working with concrete images, and the step to the abstract model is very difficult for them, and should be made with care; the step back from the abstract to the concrete is still more difficult. In both steps they should be very carefully drilled. (See Miller, pp. 164-170.)

constructing models which shall throw into relief these important elements. No less great is such a man in passing back from the abstract model to the concrete facts. The student should be constantly on the look-out for evidences of these three processes in his own thinking.

EXERCISES.

1. Discuss the following :

A little boy thought that snow ought to be sweet "because the snow is white and sugar is white too."

A child of ten, having a disagreeable teacher who was short and an agreeable one who was tall, got a new teacher who was short, and expected him to be disagreeable too.

The mouse in La Fontaine's fable, describing the cat to its mother, as a nice animal it would like to know, said, "I believe it would be very friendly towards us, for its ears are the same shape as ours." (Queyrat, *La Logique chez l'Enfant.*)

2. Why is it that in some cases a single observation is sufficient to enable us to generalise, while in other cases it is dangerous to generalise even from a large number of observations? *E.g.* All swans are white, turned out not to be true, in spite of the large number of instances which seemed to support it. But in observing in one case that when two pieces of ice are rubbed together heat results, I conclude at once that heat will always result when two pieces of ice are rubbed together.

3. Explain why some students can prove a proposition if allowed to use an acute-angled triangle, but not if compelled to use an obtuse-angled one.

4. "If you push a tablecloth with your fingers, you will find that it wrinkles up into parallel folds. The hard, solid crust of the earth has been wrinkled up in a similar manner by gigantic thrusts acting very slowly and gradually." (Mort, p. 2.)

Is this illustrating by analogy? Distinguish between illustrating by analogy and illustrating by example.

5. Describe a method of showing that ocean currents are due to the action of wind on the surface of the water. Bring out the use of the principle of analogy.

6. Illustrate the value of an abstract model in dealing with a concrete problem by showing the use (*a*) of a map in deciding as to what route to take, (*b*) of a foot-rule in deciding whether a room is large enough for a particular purpose. Give other illustrations.

SUGGESTIONS FOR READING.

Tyndall, *Glaciers of the Alps*, Everyman Edn., pp. 247-252.

CHAPTER XIII.

THE SEARCH FOR THE IMPORTANT ELEMENTS.

THE FRAMING OF HYPOTHESES.

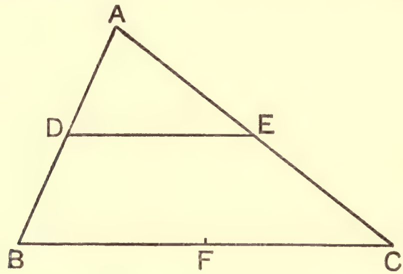
1. As we have seen in the last chapter, before we can proceed in our search for the cause of an event, we must have some general idea as to how it probably happened—as to the important elements in the case. For example, in the instance we gave as to the listlessness of children, the way seemed fairly clear when we noticed that the children were generally listless at this time of day. For then the possible causes were limited to things which might be expected to be present at such an hour. There might still be several: lack of ventilation, the nature of the lesson, fatigue due to continued strain in the previous lesson, etc. The statement of the possibilities is what is known as the framing of hypotheses. Each possibility must then be tested by itself—*e.g.* by the methods of agreement and difference, etc. If the various hypotheses cannot be completely tested, so that we cannot with certainty distinguish the true one, we must be contented with the most likely one, or the most simple one, or

the one which leads us to the most fruitful results in practice. Framing a hypothesis is not mere guessing; it involves a large amount of knowledge of the nature of the event we are investigating. We do not even trouble to exhaust all the possibilities. Some are neglected entirely, because if we did think of them we should at once reject them as absurd. For example, when a person comes into a building wearing a wet mackintosh and carrying a wet umbrella, we do not at all consider any other possibility than that it is, or has been raining. Yet there are various other ways in which a person might get wet. Now, in some cases it is dangerous to neglect possibilities which appear absurd; and an investigator (whatever the field) should never neglect them entirely. But this great tendency to neglect certain possibilities shows that all our framing of hypotheses proceeds on the basis of a fairly large knowledge of the event and its antecedents.

2. This methodical search among the various possibilities we have already spoken of as of the essential nature of thinking. We shall be helped here by a short consideration of the search for methods of solving problems in other fields than that of the search for causes.

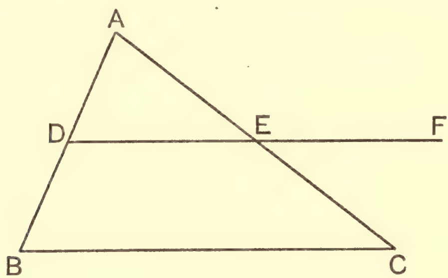
Suppose we are required to show that the line joining the mid points of the sides of a triangle is equal to half the base. What are the possible ways of proving it? In the first place we may remember that inequalities in Geometry are *all* proved ultimately by means of equalities. This is a *principle* and allows us to make the first step. We must get

on the figure a line which is half *BC* and equal to *DE*, or a line which is half *DE* and equal to *BC*. Suppose we try the first method: our most direct way would



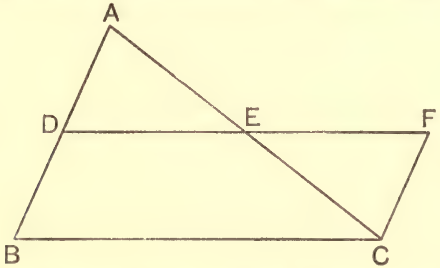
be to bisect *BC*, and try to prove *DE* equal to *BF* or *FC*. An examination of the various possible ways of doing this does not show any hope of success.

We must then try the second method. Our most direct way here would be to produce *DE*, making *EF* = *DE*. Our object is then to show *DF*



equal to *BC*. Now to prove one line equal to another there are several standard methods: either we must prove both equal to a third (which we do not seem to want here), or we must get two triangles each of which contains one of the lines, and prove these

equal (which here seems to be out of the question), or we must show that they are the opposite sides, or the diagonals of a parallelogram. Now an examination of the figure suggests that they may be opposite



sides of a parallelogram ; for if we join CF we have at once, by equality of triangles, AD equal and parallel to CF, and thus also, DB equal and parallel to CF. Thus $DBCF$ is a parallelogram ; and our result follows.

3. We see here an illustration of the kind of knowledge which is required in the successful solution of a problem. It is not merely a knowledge of *facts* : but rather a knowledge of facts as *principles.* Corresponding to the *fact* that the opposite sides of a parallelogram are equal, is the *principle* that you can prove two lines equal if you can show that they are opposite sides of a parallelogram. Corresponding to the fact that if equals be added to equals the wholes are equal, is the principle that you can prove two lines or angles or areas equal if you can break up each into parts which can be shown to be equal.

This knowledge is only to be obtained by methodical work in the solution of problems. A good

student usually tries other possible methods after he has solved the problem by one method ; he thus obtains an insight into possibilities, which is of immense value in future work. The condition of successful work in any field is an insight into possibilities, and the methodic dealing with one possibility after another.

4. The same holds in Algebra. The *fact* that a^3+b^3 can be factorised becomes the *principle* that an expression can be factorised if it can be thrown into the form of the sum of two cubes. A collection of such facts gives us a collection of principles for factorising. That student is most successful for whom these principles suggest possibilities in the treatment of a particular problem. We saw precisely the same thing in dealing with definition. Definitions from one point of view are statements of fact ; from another point of view they are principles for discovering the nature of objects. We must always be alive to this double aspect.

It should be noticed that the whole aim of setting problems to a student is that of turning facts into principles ; and that the whole object is lost if the problems are done mechanically.

5. What we have shown in regard to Algebra and Geometry is equally true of all investigation. The only useful kind of hypothesis is that in which *facts* are made use of as principles in giving the possible interpretations of the event under consideration. This is borne out by Huxley's account of the conditions to be satisfied by a good hypothesis : "We must, in the first place, be prepared to prove that the supposed causes of the phenomena exist in

nature ; that they are what the logicians call *verae causae*—true causes ; in the next place, we should be prepared to show that the assumed causes of the phenomena are competent to produce such phenomena as those which we wish to explain by them ; and in the last place, we ought to be able to show that no other known causes are competent to produce these phenomena. If we can succeed in satisfying these three conditions we shall have demonstrated our hypothesis ; or rather, I ought to say, we shall have proved it as far as certainty is possible for us." [1]

In order to satisfy the first two of these conditions it is clear that we must make use only of principles which have been given to us by facts. Thus, the fact that too little or too much water is bad for a plant, while the right amount promotes its growth, furnishes us with a legitimate principle for the formation of a possible hypothesis regarding the bad growth of a plant : perhaps its water supply was not right. The last condition must be satisfied before we can decide between the various possible hypotheses formed as above ; we must not rest until we have proved that one, and one only, is competent to produce *just* the phenomena in question. If it is impossible to decide finally, we must, as we have said, be content provisionally with the simplest, or the most useful.

The second of the conditions stated by Huxley has been put by him even more forcibly as follows : " Every hypothesis is bound to explain, or, at any rate, not to be inconsistent with, the whole of the facts it professes to explain or account for ; and if

[1] *Essays* (Everyman Edn.), p 247.

there is a single one of these facts which can be shown to be inconsistent with (I do not merely mean inexplicable by, but contrary to) the hypothesis, the hypothesis falls to the ground—it is worth nothing. One fact with which it is positively inconsistent is worth as much, and is as powerful in negativing the hypothesis, as five hundred."[1]

This is the aspect which is emphasised in criticising a hypothesis with a view to testing its truth. The scientist does not rest content with mere observation, but wherever possible, devises testing experiments. If such and such a hypothesis is the true one, then a certain result would follow on doing such and such. If such another hypothesis is true, this result will not follow.

EXERCISES.

1. A child of two, fed on milk from a white cow, said, "The milk is white because the cow is white." (Queyrat, p. 74.) Discuss the processes in the child's mind leading to this hypothesis.

2. Discuss the following:

> "A teacher giving a lesson to a young class on a bluebottle asked how the creature made its familiar buzzing noise. When she received the answer, she told the children that she expected that answer. Of course, they thought the bluebottle buzzed with its mouth, because when they wanted to buzz

[1] *Essays* (Everyman Edn.), p. 255.

they buzzed with their mouths. Accepting the teacher's word that they were wrong, the class had no peace until she told them that the buzzing was caused by the wings. This gave the children perfect satisfaction, as it did the teacher, till her Normal Master pointed out that if you remove the blue-bottle's wings it does not stop buzzing, but actually buzzes a little harder than usual. It was now the teacher's turn to be worried, and it was not till she had learned about the special little buzzing organ that she could drop the subject and be at peace once more." (Adams, p. 76.)

Do the first two suggestions satisfy any of the conditions of a good hypothesis ?

3. Does the hypothesis that the sheep has wool because it was created for man's use, and man needs wool for clothing, satisfy the conditions of a good hypothesis ?

SUGGESTIONS FOR READING.

Tyndall, *Glaciers of the Alps*, Everyman Edn., Ch. I.

CHAPTER XIV.

THE PRINCIPLE OF CAUSALITY.

We have now discussed in some detail the methods involved in forming the concepts and generalisations of ordinary life, and in investigating the reasons or causes of events. Will our methods show us that every event has a cause? There are a great many events whose cause we have not yet discovered: but does this entitle us to say whether these events have causes or not? People sometimes speak of events as due to chance or accident. This does not mean that these events have no cause. It indicates rather a certain complexity in the cause. When certain conditions are produced, bodies will act in a certain way; and in the absence of these conditions, the bodies will not act in this way. That is a statement which applies to every event, to the ticking of a clock, the fall of a stone, the motion of the ripples on the shore, the wanderings of a comet, the passing of a thought through the mind. All, we firmly believe, are produced when certain conditions are present; and if the conditions are not present, the events are not produced. Science,

as we have seen, investigates these conditions; and if it is in our power to bring about the conditions, we can produce the event. Having discovered the conditions under which the pressure, volume, and temperature of a gas vary, we can produce, *e.g.*, any given alteration of volume by altering the temperature or the pressure. Having discovered the conditions under which a person contracts an infectious disease, we can, if these conditions are within our power, take steps to prevent them being present, and so guard against the spread of infectious diseases. The old superstition which made men sacrifice to their gods in order to avert plague and pestilence has given place to a careful investigation into the natural conditions under which such events have taken place. This belief as to the way in which all events happen has been indicated by the use of the phrase "The Reign of Law in the Universe."

We believe, then, in the principle that every event has been produced under definitely definable conditions, that when these conditions are present the event will be produced, and that unless these conditions are present the event will not be produced. This principle is called the Principle or Law of Causality.

Before going on to ask whence our belief in this law comes, let us look at it a little more closely.[1]

[1] The following account of the Principle of Causality has particular reference to the use of the principle in physical science. How far it applies to human action—*e.g.* to the determination of causes in History—is a difficult question, which we do not here discuss.

(*a*) Certain sets of conditions are *isolated* from the rest of the events of the universe, in the sense that we attend to them only, leaving out of consideration all other happenings. The pressure, volume, and temperature of a gas depend on one another in such a way that, while the scientist is working in his laboratory, a battle may be going on outside; the walls may be crumbling around him; the carnage of war may be going on before his eyes;—this can safely be neglected by him as a scientist: if these things do not interfere with the pressure and temperature of his gas, they will not interfere with its volume. Indeed, he does habitually neglect changes, which if not as striking, are yet as real and as great.

In this sense the scientist can say as a result of his investigations, that provided certain conditions are present certain events will result, whatever be the state of the rest of the universe. The principle of analogy, as we have seen, depends for its successful application on our choosing those sets of conditions which *can* be isolated.

(*b*) From one point of view, an event never happens twice in the same way. There are no two sunsets exactly alike; no two leaves of a plant or tree which do not present some differences. We speak of things as being "as like as two peas"; but even two peas are different in many ways. There is in nature a wonderful variety and change. Everything under the sun is new.

But all this variety is produced by the operation of the same laws as operated before the first animal "rose from the sea"; and from this point of view, "there is no new thing under the sun." Thus,

when we speak of nature acting in uniform ways, we are thinking of the manner in which, or the conditions under which, all the variety is produced. Precisely what is meant, we have already seen in our statement of the law of causation. *There are sets of conditions to be discovered, which, whenever present* (*attended with whatever variety in nature you please*), *have the same results*; whether the time be now, or in the distant past, or in the distant future. *There are sets of conditions to be discovered*: that is the work of science. Not any set of conditions we may happen to light on at first glance, will produce the same result. But every event in the universe receives its explanation by being related to a definite set of conditions of this nature.

Whence comes our confidence in this law? It could only be proved, when we had succeeded in completely explaining every fact in the universe by its means. Even if we could explain all present facts by its means, by what right do we expect that the future course of the world will bear out our law? How can we say anything about what has not yet happened? Must we not wait and see?

Now although the human mind is capable of a strange confidence in beliefs for which there is often very little foundation, still we feel that this belief of ours is not like this. And yet it seems strange that we should have so much confidence in a belief which, by the nature of the case, we cannot prove. But perhaps on closer examination, it will not seem so strange. For if we cannot prove it, neither can we disprove it. If the scientist met with a case which seemed to show that the same conditions

were producing a different result, he would say, not that the causal law was broken, but that there were in this case modifying factors which were as yet hidden: *i.e.* that the conditions formulated were not the complete conditions affecting the result.

In the second place, if the scientist did not act in this way he would never know when to sit down with folded hands, and say, "This case shows that there is no law," or when to say, "This case presents new features which we must investigate further." In other words, it is only because the scientist believes in the law that he goes on investigating. And his faith is generally rewarded: if not in his own lifetime, then in those of his successors. Every scientist will tell you that there are many things he does not understand—*i.e.* of which he has not yet discovered the conditions; but none will tell you that these things are exceptions to, or breaches of, the uniformity of nature. Thus the law of causality is presupposed by man in all his investigations. He says to nature, "I shall regard myself as unsatisfied until I have seen the same laws operating in your new activities as I saw in your old; I demand that you shall satisfy me in this thing": and nature replies by opening up her secrets. There is then a difference between an ordinary scientific hypothesis and this principle; for all other hypotheses rest on this, and presuppose it. As it is the basis of our thinking, so it conditions our action. Unless we could count on things happening in certain definite ways, we should not act at all.

We have now answered the questions asked on p. 96, as to the methods by which we arrive at our

generalisations, and as to what proof we have for them. All our proof is based ultimately on the principle of Causality. This principle we can neither prove nor disprove ; but we cannot do without it. The significance of this fact is one of the questions discussed by Philosophy.

CHAPTER XV.

THE GENERAL NATURE OF EXPERIENCE.

Fact and theory are often contrasted, as if facts were more real than theories, things which cannot be got over, while theories are transitory things, an object of disdain to the so-called practical person. Let us see the relation between facts and theories.

We have discussed the way in which the child's experience is gained, and have seen that the process is exactly the same as that by which the scientist gains his. What is the significance of this for the present question? This: that the child is an investigator, theorising; what he theorises about is fact for the adult. The child's generalisation (or theory) that when the clock struck one it was half-past an hour, had to be tested, and when corrected, might be described as a theory justified by the facts. For the adult the part played by theory has dropped out, and for him it is a fact, and no theory, that the clock strikes in a certain way. What is a theory for the learner is a fact for the expert. Fact is thus successful theory.

Is there any fact which was never theory, *i.e.* which was at once perceived, without error or

reasoning on the part of someone? Notice that the expert will "interpret," *i.e.* form a theory about, certain events, without apparent effort, and without making a mistake. His "theory" is "fact" right off. But it is none the less theory, that is, a hypothesis as to the most probable interpretation of certain signs presented to him. The inexpert inquirer, whose interpretations are tentative, is our best guide as to how much is fact and how much theory, for in his case we see the processes more clearly at work. And we must say, that there is no fact which was not, once upon a time, seen but dimly, theorised about, and finally correctly interpreted.

Which then is the more correct statement, "Theory attempts to account for the facts," or, "Theory attempts to get at the facts"? The first suggests that we have facts complete, and then work on them: the second, that we have not the facts truly until we have understood them. It will be seen that the latter statement is the truer. Again, "These are the facts: how account for them?" would be better expressed, "These are the bare facts (or skeletons of fact): what are the real facts?" This explains why it is that apparently the same facts give rise to conflicting theories: the "facts" in question being only fragments and not complete facts. It is the man whose theory is correct who has the facts correctly; it being a law of our knowing that we should receive fragments, and build them up into real facts by our reasoning. Nature, by herself, tells us nothing: she aids us to find out things.

An example may help us to realise what is meant by partial or incomplete facts being theorised about. The child investigating the noise of the clock will serve here. That the noises he hears come from objects in certain positions in space was once a theory, and finally, tested theory or fact. That the noise he hears comes from the particular object, the clock, was once theory, and is now fact. It is less fragmentary fact than it was when the noise was less localised (*i.e.* it is more fully interpreted): but it is still fragmentary, not completely interpreted. Thus the "partial facts" of which we have been speaking are the incomplete interpretations of the material of our experience. Not incomplete in the sense of untrue: for that the noise comes from the clock is true, being tested theory; but incomplete in the sense that they have a further significance which has not yet been discovered. In just this sense the "fact" indicated by "I hear a sound" is incomplete. The stages of completeness are "I hear a sound": "It comes from the clock": "It is the clock striking the half-hour." We are adding meaning or significance, getting more fact, as we pass from one to the other.

Is all our experience then, all we know, mere theory, you may ask. Not mere theory; there is no such thing. But everything I know about—the houses, trees, birds, other people, my own actions—entered originally into my experience in a fragmentary way; I had to interpret it—theorise about it—until in the end I found a fairly consistent explanation for it all: acting on my theories, I found my expectations justified: my theories turned out to

be real facts. But man is so prone to accept his own hastily formed theories about the fragmentary facts which come before him, that many a theory is formed and believed in without sufficient justification: and it is these hasty theories with which the "practical man" is confronted every day, which give him such a distaste of "mere theory."

Thus my theory depends on the way in which facts present themselves to me. This explains how it is that so many apparent arguments turn out on examination to contain no formal reasoning. (Cf. p. 86).

The correct theory of the way in which a thing happened is thus a precise and accurate account of the facts of the case: and hence science is an attempt to get at the real facts about the universe. The best scientific theory is simply the most complete available account of the facts.

We are now in a position to understand what Herbert Spencer means when he speaks of Education as leading "from the empirical to the rational." By "the empirical" we may understand "the facts," and for "the rational" we can substitute "the theory of the facts," provided we remember the result we have reached as to the precise relation between fact and theory. The phrase will then mean, the process from the partial and fragmentary facts presented at first, to the complete and detailed account which views them in their relations to one another. Education leads us from fragmentary to complete facts.

But side by side with this phrase, Spencer uses another, in speaking of Education as leading "from

the concrete to the abstract": as if the empirical were the concrete, and theory, the rational, were the abstract. We must, however, challenge this. If we remember the process by which the scientist simplifies the conditions prevailing in nature and studies the broad lines of his subject, as it were, we shall see that the process from the detailed fragments of fact, to the abstract formulation of the law regarding the fact, is only a part of what the scientist does. His ultimate goal is to get at the complete facts. Thus he passes rather from the fragmentary concrete (if you keep to the word concrete whenever you touch fact) through the abstract to the complete concrete.

Indeed one philosopher, Hegel, working from the etymological meaning of the word abstract ("cut off") asserted, that wherever you separate from one another things which ought to be viewed in relation, there you have the abstract. Thus fragments of fact would be facts seen abstractly; the only concrete things would be facts viewed in their completeness; and he therefore said that Education leads from the abstract to the concrete.

EXERCISES.

1. Distinguish between the "empirical" and the "rational" teaching of a language. Are the names appropriate?

2. Trace the steps by which a pupil arrives at the rules:

(*a*) To divide by a vulgar fraction invert the divisor and proceed as in multiplication.
(*b*) To find the fourth term of a proportion multiply the second and third and divide by the first.

Bring out the part played by the "abstract model," and its relation to the concrete.

3. Various children were asked the question, "How do you know that such a man as George Washington ever lived?" Among the answers received were: (*a*) Because I have heard things about things that he did. (*b*) Because I saw a picture of him. (*c*) Because it was in the geography. (*d*) Because he fought bravely in many battles. Besides, he never told a lie. (*e*) Because if he didn't they wouldn't celebrate his birthday. (Earl Barnes: *Studies in Education*, 1896-7, p. 83).

Discuss the value of these answers. What answer would you give?

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